

MATHEMATICAL SCIENCES

Name & Signature of the Invigilator

PAPER-II
OCT-14/01

ICR Answer Sheet No. :

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Roll No. :

Roll Number in words :




Time : 1.15 Hours]

No. of Printed Pages : 24

[Maximum Marks : 100

Instructions for the Candidates

- Write your Roll Number in the space provided on the top of this page.
- This paper consists of fifty (50) multiple choice type questions. All questions are compulsory.
- At the commencement of examination, the question booklet will be given to candidate. In the first 5 minutes, candidate is requested to open the booklet and compulsorily examine it as below :
 - To have access to the question booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal and do not accept an open booklet.
 - Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of five minutes. Afterwards, neither the question booklet will be replaced nor any extra time will be given.
 - After this verification is over, the test booklet number should be entered in the ICR answer sheet and the ICR Answer Sheet number should be entered on this test booklet.
- Each item has upto four alternative responses marked (A), (B), (C) and (D). The answer should be a capital letter for the selected option. The answer letter should entirely be contained within the corresponding square.

Correct method  Wrong method  OR 
- Your responses to the items for this paper are to be indicated on the ICR Answer Sheet under Paper II only.
- Read instructions given inside carefully.
- Rough work is to be done in the end of the booklet only.
- You have to return the original ICR Answer Sheet to the invigilators at the end of the examination compulsorily and must not carry it with you outside the examination hall. You are, however, allowed to carry duplicate copy of ICR sheet and test booklet on conclusion of the examination.
- Use black ball point pen.
- Use of any Calculators or log tables or any other electronic devices is prohibited.
- There shall be no negative marking.
- In case of any discrepancy in Gujarati and English version of questions the English version should be taken as final.

પરીક્ષાર્થીઓ માટે સૂચનાઓ :

- આ પાનાની ટોચમાં દર્શાવેલી જગ્યામાં તમારો રોલનંબર લખો.
- આ પ્રશ્નપત્રમાં બહુવૈકલ્પિક ઉત્તરો ધરાવતા કુલ પચાસ (૫૦) પ્રશ્નો આપેલા છે. બધા જ પ્રશ્નો ફરજિયાત છે.
- પરીક્ષાની શરૂઆતમાં ઉમેદવારને પ્રશ્નપુસ્તિકા આપવામાં આવશે. પ્રથમ ૫ મિનિટ દરમિયાન, ઉમેદવારે પ્રશ્નપુસ્તિકા ખોલી અને ફરજિયાતપણે નીચે મુજબ પરીક્ષણ કરવું.
 - પ્રશ્નપુસ્તિકાનો વપરાશ કરવા માટે આ કવર પેજની ધાર પર આપેલ સીલ ફાડી નાખો. કોઈપણ સંજોગોમાં સીલ સ્ટીકર વગરની કે ખુલ્લી પ્રશ્નપુસ્તિકા સ્વીકારશો નહીં.
 - કવર પૃષ્ઠ પર છપાયેલ નિર્દેશાનુસાર પ્રશ્નપુસ્તિકાના પ્રશ્નો પૂઠ્ઠો અને સંખ્યાને બરાબર ચકાસી લો. ખામીયુક્ત પ્રશ્નપુસ્તિકા કે જેમાં પૂઠ્ઠો/પ્રશ્નો ઓછા હોય, બે વાર છપાયા હોય, અનુક્રમમાં અથવા કોઈ અન્ય ફરક હોય અર્થાત કોઈપણ કારણે ખામીયુક્ત પ્રશ્નપુસ્તિકા સ્વીકારવી નહીં. એને જો ખામીયુક્ત પ્રશ્નપુસ્તિકા મળી હોય તો નિરીક્ષક પાસેથી તુરંત જ બીજી સારી પ્રશ્નપુસ્તિકા મેળવી લેવી. આ માટે ઉમેદવારને પાંચ મિનિટનો સમયગાળો આપવામાં આવશે. પછીથી, પ્રશ્નપુસ્તિકા બદલવામાં આવશે નહીં કે કોઈ વધારાનો સમય પણ આપવામાં આવશે નહીં.
 - આ ચકાસણી સમાપ્ત થાયપછી, ટેસ્ટ પુસ્તિકા નંબર ICR જવાબ પત્રકમાં લખવો અને ICR જવાબ પત્રક નંબર પ્રશ્નપુસ્તિકા પર લખવો.
- પ્રત્યેક પ્રશ્ન માટે ચાર ઉત્તર વિકલ્પ (A), (B), (C) અને (D) આપવામાં આવેલ છે. પસંદગીનો જવાબ માત્ર અંગ્રેજી કેપીટલ મૂળાક્ષર દ્વારા જ આપવો. પસંદ કરેલ અંગ્રેજી કેપીટલ અક્ષર આપેલ ખાનામાં સંપૂર્ણ રીતે સમાઈ જાય તે રીતે લખવો.

સાચી રીત :



ખોટી રીત :



અથવા



- આ પ્રશ્નપુસ્તિકાના પ્રશ્નોના જવાબ અલગથી આપવામાં આવેલ ICR જવાબ પત્રકમાં પેપર-૨ લખેલ વિભાગમાં જ લખવા.
- અંદર આપેલ સૂચનાઓ ધ્યાનપૂર્વક વાંચો.
- આ પ્રશ્નપુસ્તિકાની અંતે આપેલ પાનું રફ કામ માટે છે.
- પરીક્ષા સમય પૂરો થઈ ગયા પછી ઓરીજનલ ICR જવાબ પત્રક જે તે નિરીક્ષકને ફરજિયાત સોંપી દેવું અને કોઈપણ સંજોગોમાં પરીક્ષાખંડની બહાર જઈ શકશે નહીં. પરીક્ષા પૂર્ણ થયા બાદ ઉમેદવાર પ્રશ્નપુસ્તિકા તથા ICR જવાબવહીની ડુપ્લિકેટ કોપી પોતાની સાથે લઈ જઈ શકે છે.
- માત્ર કાળી પેન/કાળી બોલ પેન વાપરવી.
- કેલ્ક્યુલેટર અને અન્ય ઈલેક્ટ્રોનિક યંત્રોનો ઉપયોગ કરવાની મનાઈ છે.
- ખોટા જવાબ માટે નેગેટિવ ગુણાંકન પ્રથા નથી.
- પ્રશ્નપુસ્તિકાના કોઈ પ્રશ્નમાં અનુવાદ અંગે કોઈ વિવાદ/મતભેદ જણાય તો અંગ્રેજી વર્ઝન યોગ્ય ગણાશે.

LOGARITHMS

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Mean Differences | | | | | | | | |
|----|------|------|------|------|------|------|------|------|------|------|------------------|---|----|----|----|----|----|----|----|
| | | | | | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 | 0212 | 0253 | 0294 | 0334 | 0374 | 4 | 8 | 12 | 17 | 21 | 25 | 29 | 33 | 37 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 | 4 | 8 | 11 | 15 | 19 | 23 | 26 | 30 | 34 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 | 3 | 7 | 10 | 14 | 17 | 21 | 24 | 28 | 31 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 | 3 | 6 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 | 3 | 6 | 8 | 11 | 14 | 17 | 20 | 22 | 25 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 | 3 | 5 | 8 | 11 | 13 | 16 | 18 | 21 | 24 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 | 2 | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 22 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 | 2 | 4 | 7 | 9 | 11 | 13 | 16 | 18 | 20 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 | 2 | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 22 | 3424 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 15 | 17 |
| 23 | 3617 | 3636 | 3655 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3784 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 | 2 | 4 | 5 | 7 | 9 | 11 | 12 | 14 | 16 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 | 2 | 3 | 5 | 7 | 9 | 10 | 12 | 14 | 15 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 | 2 | 3 | 5 | 7 | 8 | 10 | 11 | 13 | 15 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 13 | 14 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 29 | 4624 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 12 | 13 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 11 | 13 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 12 |
| 32 | 5051 | 5065 | 5079 | 5092 | 5105 | 5119 | 5132 | 5145 | 5159 | 5172 | 1 | 3 | 4 | 5 | 7 | 8 | 9 | 11 | 12 |
| 33 | 5185 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 | 1 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 12 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 | 1 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 11 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5514 | 5527 | 5539 | 5551 | 1 | 2 | 4 | 5 | 6 | 7 | 9 | 10 | 11 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 5635 | 5647 | 5658 | 5670 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 | 1 | 2 | 3 | 4 | 5 | 7 | 8 | 9 | 10 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 44 | 6435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 45 | 6532 | 6542 | 6551 | 6561 | 6571 | 6580 | 6590 | 6599 | 6609 | 6618 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 8 |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 49 | 6902 | 6911 | 6920 | 6929 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 7 |
| 53 | 7243 | 7251 | 7259 | 7267 | 7275 | 7284 | 7292 | 7300 | 7308 | 7316 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| 54 | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |

No.
 $\pi = 3.14159$
 $e = 2.71828$

log
 0.49715
 0.43429

$\ln x = \log_e x = (1/M) = \log_{10} x$
 $\log x = \log_{10} x = M \log_e x$

No.
 $(1/M) = 2.30259$
 $M = 0.43429$

log
 0.36222
 1.63778

MATHEMATICAL SCIENCES

PAPER-II

Note : This paper contains **FIFTY (50)** multiple-choice/matching questions, each question carrying **TWO (2)** marks. Attempt **All** the questions.

1. Let (a_n) and (b_n) be sequences of real numbers such that $a_n > 0$ and

$$b_n > 0 \forall n \in \mathbf{N}. \text{ Let } c_n = \frac{a_n}{b_n} \text{ for } n \in \mathbf{N}.$$

If (a_n) converges to a and (b_n) converges to b , then :

(A) (c_n) is convergent

(B) (c_n) is convergent only if $b > 0$

(C) $\limsup_{n \rightarrow \infty} c_n = \infty$ if (c_n) is not convergent

(D) (c_n) is always bounded

2. Let $a_n = \sqrt{n+1} - \sqrt{n}$ and $b_n = \frac{\sqrt{n+1} - \sqrt{n}}{n}$ for $n \in \mathbf{N}$.

Then :

(A) $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are both convergent

(B) $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are both divergent

(C) $\sum_{n=1}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} b_n$ is divergent

(D) $\sum_{n=1}^{\infty} a_n$ is divergent but $\sum_{n=1}^{\infty} b_n$ is convergent

3. The sequence $\{f_n\}$, where $f_n(x) = \frac{1}{nx+1}$, $n \geq 1$, converges :

- (A) Uniformly on $[0, 1]$
- (B) Uniformly on $(0, \infty)$
- (C) Uniformly on $[a, \infty)$ where $a > 0$
- (D) Uniformly on $(0, a]$, where $a > 0$

4. Let $f : \mathbf{R}^2 \rightarrow \mathbf{R}$ be given by :

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

Then :

- (A) $f_x(0, 0)$ and $f_y(0, 0)$ exist but f is not continuous at $(0, 0)$
- (B) f is differentiable at $(0, 0)$
- (C) f is continuous at $(0, 0)$ but f is not differentiable at $(0, 0)$
- (D) Both the partial derivatives f_x and f_y are continuous at $(0, 0)$

5. The equation $e^x = \sin x$ has :

- (A) No real solution
- (B) Exactly one real solution
- (C) Finitely many solutions in $(-\infty, 0)$
- (D) Infinitely many real solutions

6. The value of

$$\lim_{n \rightarrow \infty} \left\{ \frac{1}{n} \sum_{k=0}^{n-1} \sin\left(\frac{k}{n}\right) \right\}$$

is :

(A) 1

(B) 2

(C) 0

(D) $1 - \cos 1$

7. Let A and B be connected subsets of \mathbf{R}^2 with $A \cap B \neq \phi$, then :

(A) A° , \bar{A} and $A \cup B$ are connected

(B) A° and $A \cap B$ are connected

(C) \bar{A} and $A \cup B$ are connected but $A \cap B$ and A° may not be connected

(D) \bar{A} , $A \cup B$ and $A \cap B$ are connected but A° may not be connected

8. Let V and W be real vector spaces and $T : V \rightarrow W$ be a linear transformation.

Consider the following statements :

(i) If $\{V_1, \dots, V_n\}$ is a linearly independent subset of V, then $\{TV_1, \dots, TV_n\}$ is a linearly independent subset of W.

(ii) If $\{TV_1, \dots, TV_n\}$ is a linearly independent subset of W, then $\{V_1, \dots, V_n\}$ is a linearly independent subset of V.

Then :

(A) both (i) and (ii) are true

(B) (i) is true but (ii) is false

(C) (ii) is true but (i) may not be true

(D) both (i) and (ii) are false

9. Let V be the set of real function $y = f(x)$ satisfying

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0.$$

Then $\dim \text{Hom}(V, \mathbf{R}) =$

- (A) 3 (B) 4
(C) 2 (D) 5

10. In the vector space $C[a, b]$; $a, b \in \mathbf{R}, a < b$, :

- (A) $\{x^3, x^4\}$ is linearly independent but $\{\sin x, \cos x\}$ is not
(B) $\{x^3, \sin x \cos x\}$ is linearly independent but $\{x^3, x, \sin x\}$ is linearly dependent
(C) $\{x^3, x^4, \sin x\}$ is linearly independent
(D) $\{x^3, x^4, \sin x, \cos x\}$ is linearly dependent

11. Which of the following is *not* a basis for the vector space of all real polynomials of degree $\leq n$?

- (A) $1, x, x^2, \dots, x^n$
(B) $1, 1 - x, (1 - x)^2, \dots, (1 - x)^n$
(C) $x, x^2, x^3, \dots, x^n, 1 + x^n$
(D) $x, x^2, x + x^2, x^3, \dots, x^{n-1}, 1 + x^n$

12. Let A be a $m \times n$ matrix. Then the space of solutions of the linear system $AX = 0$ has dimension at least :

(A) $m + n$

(B) m, n

(C) $n - m$

(D) 1

13. The diagonalization of the matrix $\begin{bmatrix} 2 & i \\ -i & 2 \end{bmatrix}$ yields :

(A) $\begin{bmatrix} 1 & \\ & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & \\ & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & \\ & 2 \end{bmatrix}$

(D) $\begin{bmatrix} 3 & \\ & 1 \end{bmatrix}$

14. Let V be a finite dimensional vector space, S be a subset of V that spans V and L is a linearly independent subset of V . Then :

(A) $|S| = |L| = \dim V$

(B) $|S| = |L|$ if L also spans V

(C) $|S| = |L|$ if S is also linearly independent

(D) $|S| \geq \dim V \geq |L|$

15. The function $\log(\sin 2\pi z)$ has :
- (A) A simple pole at $\frac{1}{2}$
 - (B) A removable singularity at $\frac{1}{2}$
 - (C) An essential singularity at $\frac{1}{2}$
 - (D) A branch point at $\frac{1}{2}$
16. The function $\cos z$ maps :
- (A) Horizontal lines other than x -axis to parabolas
 - (B) Horizontal lines other than x -axis to ellipses
 - (C) Vertical lines other than y -axis to ellipses
 - (D) Vertical lines other than y -axis to parabolas
17. Let $f(z) = z^3 + 3z = u(x, y) + iv(x, y)$.

Then the value of $\det \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix}$ at $z = 1$ equals :

- (A) 24
- (B) 28
- (C) 32
- (D) 36

18. If $f(z) = \frac{(2 - e^z - e^{-z})^k}{z^7}$ has a simple pole at the origin, then the value of k is :

- (A) 1 (B) 2
(C) 3 (D) 4

19. The residue of $\frac{\cos z}{z^4 - 1}$ at $z = i$ equals :

- (A) $\frac{i}{4} \cosh 1$ (B) $\frac{i}{4} \cos 1$
(C) $\frac{i}{4} \sin 1$ (D) $\frac{i}{4} \sinh 1$

20. Which of the following polynomials is irreducible ?

- (A) $x^4 + 1 \in \mathbf{Z}_2[x]$
(B) $x^4 + 1 \in \mathbf{R}[x]$
(C) $x^{10} + x^9 + x^8 + \dots + x + 1 \in \mathbf{Q}[x]$
(D) $x^5 - 2 \in \mathbf{C}[x]$

21. Let G be a group of order 200. Then :

- (A) G has a normal Sylow 5-subgroup
(B) G is simple
(C) G is abelian
(D) G is not solvable

22. If $\sigma \in S_n$ has odd order, then :
- (A) σ is an odd permutation
 - (B) σ is an even permutation
 - (C) σ is a product of an odd number of disjoint cycles
 - (D) σ is a product of an even number of disjoint cycles
23. Consider the group \mathbf{Z} of integers under addition. Then :
- (A) $\text{Aut } \mathbf{Z} \cong \mathbf{Z}$
 - (B) $\text{Aut } \mathbf{Z} \cong \mathbf{Z}_2$
 - (C) $\text{Aut } \mathbf{Z} \cong \mathbf{R}$
 - (D) $\text{Aut } \mathbf{Z}$ is a trivial group
24. The Galois group of the polynomial $x^3 - 2 \in \mathbf{Q}[x]$ is :
- (A) \mathbf{Z}_3
 - (B) S_3
 - (C) \mathbf{Z}_6
 - (D) Klein's 4-group
25. Let φ be Euler's function. Then $\varphi(2n) = \varphi(n)$ if and only if :
- (A) n is divisible by 4
 - (B) n is 1
 - (C) n is prime
 - (D) n is odd

26. $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of :

$$(1 - x^2)y'' - xy' + p^2y = 0.$$

Given that $y_1(0) = 1$, $y_1'(0) = 0$, $y_2(0) = 1$, the value of the Wronskian $W(y_1, y_2)$ at $x = \frac{1}{2}$ is :

- (A) $\frac{2}{\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2}$
(C) $\frac{1}{2}$ (D) 2

27. The characteristic curves in (x, t) plane for $tu_x - xu_t = 0$ are :

- (A) rays starting from the origin
(B) circles in the (x, t) plane with centre at the origin
(C) family of lines parallel to x -axis
(D) family of lines parallel to t -axis

28. Which of the following is *true* ?

- (A) Both the wave equation and Laplace's equation are hyperbolic
(B) Both the equations are elliptic
(C) The Laplace's equation is elliptic and the wave equation is hyperbolic
(D) The Laplace's equation is hyperbolic and the wave equation is elliptic

29. Let $(x(t), y(t))$ be a solution of the coupled system of harmonic oscillators :

$$\ddot{x} + \omega_1^2 x = 0, \ddot{y} + \omega_2^2 y = 0.$$

It is known that the parametrized curve $(x(t), y(t))$ is an ellipse. Then which of the following is *true* ?

- (A) $\omega_1^2 = 4 \omega_2^2$ (B) $7 \omega_1^2 = 3 \omega_2^2$
(C) $\omega_1^2 = 2 \omega_2^2$ (D) $4 \omega_1^2 = 3 \omega_2^2$
30. Given a (3×3) matrix A with real entries such that whenever $u(x)$ is a harmonic function of (x_1, x_2, x_3) , the function $v(x) = u(Ax)$ is also harmonic. Then, A can be :

- (A) Any (3×3) non-singular matrix
(B) Any (3×3) symmetric matrix
(C) Any (3×3) orthogonal matrix
(D) Any (3×3) matrix with determinant 1
31. The Fredholm integral equation

$$\phi(x) - \lambda \int_0^1 \cosh(x-t) \phi(t) dt = 0$$

has :

- (A) no eigen values
(B) at least one real eigen value
(C) infinitely many eigen values with a finite limit point
(D) an eigen value λ_0 corresponding to which there are infinitely many eigen vectors

32. Let \vec{r} be the position vector of a point moving under the action of a force

\vec{F} . If \vec{F} is parallel to the negative z -axis, and $\vec{L} = \frac{d\vec{r}}{dt} \times \vec{r}$, then :

- (A) The z -component of \vec{L} is constant
- (B) $|\vec{L}|$ is constant
- (C) All the components of \vec{L} are constant
- (D) None of the components of \vec{L} remain constant

33. Given distinct real numbers x_1, \dots, x_n ,

$$\text{let } L_j(x) = \frac{(x - x_1) \dots (x - x_{j-1})(x - x_{j+1}) \dots (x - x_n)}{(x_j - x_1) \dots (x_j - x_{j-1})(x_j - x_{j+1}) \dots (x_j - x_n)}$$

be the Lagrange polynomial. Let $Q(x) = x^3 + 2x + 7$.

Then $\sum_{j=1}^n Q(x_j)L_j(3)$ equals :

- (A) 40
- (B) 39
- (C) 30
- (D) 29

34. The initial value problem :

$$\frac{dy}{dx} = \sqrt{|x-1|} + \sqrt{\left| \sin\left(\frac{\pi x}{2}\right) \right|}; y(a) = 0$$

has :

- (A) a unique solution in a neighbourhood of a if $a = 0$
- (B) infinitely many solutions in a neighbourhood of a if $a = 0$
- (C) a unique solution in a neighbourhood of a if $a = 1$
- (D) finitely many solutions in a neighbourhood of a if $a = 1$

35. The integral of function $f(x) = 3|x - 1| + 2|x + 2|$ over the interval $[-1, 1]$ is computed using trapezoidal rule with partition points of step size (0.2). The difference between the computed value and actual value is :

- (A) 0.2 (B) -0.4
(C) 0 (D) -0.2

36. The ODE $y'' - y = xe^x$ is solved by the method of undetermined coefficients. The form of the particular integral is :

- (A) $Ae^x + Be^{-x}$
(B) $Axe^x + Bxe^{-x}$
(C) $Ax^2e^x + Bx^2e^{-x}$
(D) $Ax^2e^x + Bx^2e^{-x} + Cxe^x + Dxe^{-x}$

37. If the rational numbers a, b, c are such that $a + b^3\sqrt{2} + c^2\sqrt{3} = 0$, then :

- (A) $a > b = c = 0$ (B) $a > b > c = 0$
(C) $a = b = c = 0$ (D) $a > b > c > 0$

38. For children between ages of 18 months and 29 months, there is approximately a linear relationship between "height" and "age". The relationship can be represented by :

$$Y = 64.93 + 0.63X$$

where Y represents height (in cms) and X represents age (in months). Joseph is 22.5 months old and is 80 cm tall. What is Joseph's residual ?

- (A) 79.1 (B) -0.9
- (C) 0.9 (D) 56.6
39. In a single-factor ANOVA problem involving five populations, with a random sample of four observations from each one, it is found that $SST_r = 16.1408$, and $SSE = 37.3801$. Then the value of the test statistic is :
- (A) 1.619 (B) 2.316
- (C) 0.432 (D) 1.522
40. Which of the following is *not* the goal of descriptive statistics ?
- (A) Summarizing data
- (B) Displaying aspects of the collected data
- (C) Reporting numerical findings
- (D) Estimating characteristics of the population

41. The expected number of heads in 300 tosses of a fair coin is :
- (A) 300 (B) 250
(C) 200 (D) 150
42. In testing H_0 against H_1 , power function indicates :
- (A) $P[\text{Rejecting } H_0/\theta, \text{ where } \theta \text{ is a scale parameter}]$
(B) $P[\text{Rejecting } H_0/H_1]$
(C) $P[\text{Rejecting } H_0/H_0]$
(D) $1 - P[\text{Rejecting } H_0/H_1]$
43. Suppose a statistician conducts an experiment to test a hypothesis of interest. If she decides to double the sample size, which of the following will be a consequence of this ?
- (A) The size of the test will be doubled
(B) The power of the test will be increased
(C) The probability of type-2 error will be increased
(D) Both size and power will be doubled

44. A squad of 20 players for a regional team is made up of 8 from club C, 7 from club M and 5 from club L. Seven players are selected at random from these 20. The probability that 3 are from C, 2 from M and 2 from L is :

(A) $\frac{96}{324}$ (B) $\frac{98}{323}$
 (C) $\frac{49}{158}$ (D) $\frac{98}{326}$

45. Let $\{X_n\}_0^\infty$ be a doubly stochastic Markov chain such that :

$$P(X_{n+1} = 1 | X_n = 1) = p_1 = 1 - P(X_{n+1} = 0 | X_n = 1),$$

$$P(X_{n+1} = 1 | X_n = 0) = p_0 = 1 - P(X_{n+1} = 0 | X_n = 0),$$

and

$$P(X_1 = 1) = \pi_1 = 1 - P(X_1 = 0).$$

Then :

(A) $\pi_1 = p_1$ (B) $\pi_1 = p_0$
 (C) $\pi_1 = \frac{p_1 - p_0}{2}$ (D) $\pi_1 = \frac{1}{2}$

46. X is a Poisson random variable with variance equal to 1.5, which of the following statements is *false* ?

(A) $E(X^2) = 3.75$ (B) $(CV)^2$ of X is $\frac{2}{3}$
 (C) mode = 1 (D) $\frac{P(X = 2)}{P(X = 1)} = \frac{4}{3}$

47. Suppose X_1 and X_2 are independent random variables with common distribution function $F(x)$. Let $U = \min(X_1, X_2)$ and $V = \max(X_1, X_2)$, then the distribution function of U and V are respectively :
- (A) $(1 - F(x))^2$ and $(F(x))^2$
 - (B) $1 - (1 - F(x))^2$ and $(F(x))^2$
 - (C) $2(1 - F(x)) f(x)$ and $2F(x) \cdot f(x)$
 - (D) $1 - (1 - F(x))^2$ and $1 - (F(x))^2$
48. Suppose X_1 and X_2 are independent Bernoulli random variables with $E(X_1) = p_1$ and $E(X_2) = p_2$, then $X_1 \cdot X_2$ follows :
- (A) Bernoulli distribution with mean $p_1 p_2$
 - (B) Bernoulli distribution with mean $1 - p_1 p_2$
 - (C) Binomial distribution with $n = 2$ and $p = p_1 p_2$
 - (D) Binomial distribution with $n = 2$ and $p = 1 - p_1 p_2$
49. For testing $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$ based on two independent random samples of size n_1 and n_2 drawn from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively. We use the following test when σ_1^2 and σ_2^2 are unknown :
- (A) Student's t -test
 - (B) Standard normal deviate test
 - (C) Chi-square test
 - (D) Fisher-Behren's test

50. Match the following lists :

List I

List II

(i) If $X \sim X_{n_1}^2$, $Y \sim X_{n_2}^2$, then (a) Fisher's t -distribution

$\frac{X}{Y}$ follows

(ii) If $X \sim F_{(m,n)}$ then $\frac{1}{X}$ (b) $B_1\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$

follows

(iii) If $X \sim N(0,1)$ and $Y \sim \frac{X^2}{n}$, (c) $B_2\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$

then $\frac{X}{\sqrt{Y}}$ follows

(iv) If $X \sim X_{n_1}^2$, $Y \sim X_{n_2}^2$, then (d) $F_{(n,m)}$

$\frac{X}{X+Y}$ follows

The correct match is :

(A) (i)—(b), (ii)—(c), (iii)—(d), (iv)—(a)

(B) (i)—(c), (ii)—(d), (iii)—(a), (iv)—(b)

(C) (i)—(b), (ii)—(d), (iii)—(a), (iv)—(c)

(D) (i)—(c), (ii)—(a), (iii)—(b), (iv)—(d)

ROUGH WORK

ROUGH WORK

SEAL