MATHEMATICAL SCIENCE

Paper - II

OCT-10/01

Signature of Invigilators

1. ....

2. ....

Roll No. [ ][ ][ ][ ][ ]

(In figures as in Admit Card)

Roll No. ....

....

(in words)

Time Allowed : 75 Minutes] **[Maximum Marks : 100**

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page.
2. This paper consists of **fifty (50)** multiple choice type questions. **All** questions are compulsory.
3. Each item has upto four alternative responses marked (A), (B), (C) and (D). The answer should be a capital letter for the selected option. The answer letter should entirely be contained within the corresponding square.

   Correct method [A] Wrong method [A] OR [A]

4. Your responses to the items for this paper are to be indicated on the ICR Answer Sheet under Paper II only.
5. Read instructions given inside carefully.
6. Extra sheet is attached at the end of the booklet for rough work.
7. You should return the test booklet to the invigilator at the end of paper and should not carry any paper with you outside the examination hall.
8. There shall be no negative marking.
9. Use of calculator or any other electronic devices is prohibited.

પરીક્ષાર્થીઓ માટે સૂચનાઓ :

૧. આ પાનાની ટોચમાં દર્શાવેલી જગ્યામાં તમારો રોલનંબર લખો.

૨. આ પ્રશ્નપત્રમાં બહુવૈકલ્પિક ઉત્તરો ધરાવતા કુલ **પચાસ (૫૦)** પ્રશ્નો આપેલા છે. **બધા જ** પ્રશ્નો ફરજિયાત છે.

૩. પ્રત્યેક પ્રશ્ન વધુમાં વધુ ચાર બહુવૈકલ્પિક ઉત્તરો ધરાવે છે. જે (A), (B), (C) અને (D) વડે દર્શાવવામાં આવ્યા છે. પ્રશ્નનો ઉત્તર કેપીટલ સંજ્ઞા વડે આપવાનો રહેશે. ઉત્તરની સંજ્ઞા આપેલ ખાનામાં બરાબર સમાઈ જાય તે રીતે લખવાની રહેશે.

   ખરી રીત : [A] ખોટી રીત : [A] , [A]

૪. આ પ્રશ્નપત્રના જવાબ આપેલ ICR Answer Sheet ના Paper II વિભાગની નીચે આપેલ ખાનાઓમાં આપવાના રહેશે.

૫. અંદર આપેલ સૂચનાઓ કાળજીપૂર્વક વાંચો.

૬. આ બુકલેટની પાછળ આપેલું પાનું રફ કામ માટે છે.

૭. પરીક્ષા સમય પૂરો થઈ ગયા પછી આ બુકલેટ જે તે નિરીક્ષકને સોપી દેવી. કોઈપણ કાગળ પરીક્ષા ખંડની બહાર લઈ જવો નહીં.

૮. ખોટા જવાબ માટે નેગેટિવ ગુણાંકન પ્રથા નથી.

૯. કેલ્કયુલેટર અને ઈલેક્ટ્રોનિક યંત્રોનો પ્રયોગ કરવાની મનાઈ છે.

MATHEMATICAL SCIENCE

PAPER–II

Note : This paper contains **FIFTY (50)** multiple-choice/Assertion and Reasoning/Matching questions. Each question carrying **two (2)** marks. Attempt ALL the questions.

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1. Let $x_n = \frac{n^2}{\sqrt{n^6+1}} + \frac{n^2}{\sqrt{n^6+2}} + \dots + \frac{n^2}{\sqrt{n^6+n}}$. Then $\langle x_n \rangle$ converges to :

(A) 1 (B) 0

(C) $\frac{1}{2}$ (D) $\frac{3}{2}$

2. The sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$ is :

(A) $\frac{1}{6}$ (B) 1

(C) $\frac{1}{18}$ (D) $\frac{1}{2}$

3. The series $\frac{(a+x)}{1!} + \frac{(a+2x)^2}{2!} + \frac{(a+3x)^3}{3!} + \dots$ is convergent if :

(A) $x > \frac{1}{e}$ (B) $x < \frac{1}{e}$

(C) $x = \frac{1}{e}$ (D) $x > 1$

4. For a function f of several variables,

(A) if f has directional derivatives in all directions, then f is continuous

(B) if f is continuous, then all the partial derivatives of f exist

(C) if all the partial derivatives of f are continuous, then f is differentiable

(D) if f is differentiable, then all the partial derivatives of f are continuous

5. $f(x) = x^4 + 2x^3 - 2, x \in \mathbf{R}$,

(A) has two roots in $[0, \infty)$ (B) has three roots in $[0, \infty]$

(C) has no root in $[0, \infty)$ (D) has a unique root in $[0, \infty)$

6. If $f : [0, 2] \to \mathbf{R}$ is defined by $f(x) = \begin{cases} 3 & , 0 \le x < 1 \\ 9 & , x = 1 \\ 4 & , 1 < x \le 2 \end{cases}$, then :

(A) the upper integral $\overline{\int_0^2} f(x)dx = 9$

(B) the upper integral $\overline{\int_0^2} f(x)dx = 7$

(C) the lower integral $\underline{\int_0^2} f(x)dx = 6$

(D) the lower integral $\underline{\int_0^2} f(x)dx = 9$

7. The subset of $\mathbf{R}^2$, which is compact under usual metric, is :

(A) $\{(x, y) \mid xy = 1\}$ (B) $\{(x, y) \mid 0 < 3x^2 + y^2 \le 1\}$

(C) $\{(x, y) \mid y^2 = x\}$ (D) $\{(x, y) \mid 1 \le x^2 + y^2 \le 2\}$

8. If $T : \mathbf{R}^7 \to \mathbf{R}^7$ is a linear map such that $T^2 = 0$, then the rank of T is :

(A) ≤ 3 (B) > 3

(C) 5 (D) 6

9. Let A be a square matrix with entries from the set of complex numbers. Then :

(A) if A is Hermitian then the eigenvalues of A are purely imaginary

(B) if A is unitary then the eigenvalues of A are in the set $\{z : |z| = 1\}$

(C) If A is skew Hermitian then the eigenvalues of A are real

(D) If A is unitary then eigenvalues of A are in [0, 1]

10. If V is the vector space over **R**, consisting of solutions of $\frac{d^2y}{dx^2} + 9y = 0$, then dim V is :

(A) 1 (B) 2

(C) 3 (D) ∞

11. Let V be a vector space of all polynomials of degree at most 2 and with coefficients in **R**. Consider the bases :

$\mathbf{A} = \{(1 + x), x, (x + x^2)\}$ and

$\mathbf{B} = \{1, (1 + x), (1 + x + x^2)\}$ for V.

The transition matrix from **A** and **B** is :

(A) $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

12. For $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$, let the innerproduct in $\mathbf{C}^3$ be defined

by $(u, v) = (\bar{u}_1, \bar{u}_2, \bar{u}_3) A\, (v_1, v_2, v_3)^T$, where $A = \begin{bmatrix} 2 & i & -1 \\ -i & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$.

If $u = (1 - i, 3i, -2)$ and $v = (2i, 6, -i)$, then $|(u, v)|$ is equal to :

(A) $\sqrt{3721}$ (B) $\sqrt{3842}$

(C) $\sqrt{551}$ (D) $\sqrt{4143}$

13. Let $f(x, y, z) = (\sin xy) e^{-z^2}$. The direction from $(1, \pi, 0)$ for f to increase most rapidly is

(A) $i - j$ (B) $-i + j$

(C) $-\pi i - j$ (D) $\pi i - j$

14. Reflection of the line $\bar{a}z + a\bar{z} = 0$ in the real axis is

(A) $\bar{a}\,\bar{z} + az = 0$ (B) $\bar{a}\,z - a\bar{z} = 0$

(C) $(a + \bar{a})(z + \bar{z}) = 0$ (D) $az - \bar{a}\,\bar{z} = 0$

15. The value of the integral $\int_{|z|=1} \frac{2z - 1}{(z - 1)^2 (z + 2)} dz$ is

(A) $2\pi i$ (B) $\frac{2\pi i}{3}$

(C) $\frac{\pi i}{3}$ (D) $\frac{2\pi i}{5}$

16. Let $w = f(z) = z + \frac{1}{z}$, $z = x + iy, w = u + iv$, then

(A) f maps $|z| = 1, 0 \leq \theta \leq \pi$ onto $-2 \leq u \leq 2, v = 0$

(B) f map $|z| = 1, \pi \leq \theta \leq 2\pi$ onto $-2 \leq u \leq 2, v = 1$

(C) f maps $-1 \leq x < 0$ onto $-\infty < u \leq 0$

(D) f maps $0 < x \leq 1$ onto $0 \leq u < \infty$

17. The radius of convergence of $\Sigma z^{n!}$ is

(A) 2 (B) 0

(C) 1 (D) ∞

18. The principal part of Laurent series expansion of $f(z) = \frac{(z^2 + 1)}{\sin(z^3)}$ is

(A) $\frac{1}{z^3}$ (B) $\frac{1}{z}$

(C) $\frac{1}{z^3} - \frac{1}{z}$ (D) $\frac{1}{z^3} + \frac{1}{z}$

19. Which of the following is *not* true ?

(A) $f(z) = \frac{1}{z}$ has a pole at $z = 0$

(B) $f(z) = \sin\frac{1}{z}$ has essential singularity at $z = 0$

(C) $f(z) = \frac{1}{z^3} - \cos z$ has essential singularity at $z = 0$

(D) $f(z) = \frac{\sin z}{z}$ has removable singularity at $z = 0$

20. The complete list of least positive incongruent solutions of $51x \equiv 12 \pmod{87}$ is

(A) {19, 47, 67} (B) {17, 48, 68}

(C) {15, 48, 67} (D) {19, 48, 67}

21. The group $\mathbf{Q}/\mathbf{Z}$ is isomorphic to

(A) $\mathbf{Q} \times \mathbf{Q}$

(B) P, the group of roots of unity

(C) S^1, the circle group

(D) $2\mathbf{Z}$

22. Let G be a group of order 1225. Then G is

(A) non-abelian

(B) abelian and is always isomorphic to $\mathbf{Z}_{5^2} \times \mathbf{Z}_{7^2}$

(C) abelian and may be isomorphic to either $\mathbf{Z}_{5^2} \times \mathbf{Z}_{7^2}$ or $\mathbf{Z}_{5^2} \times \mathbf{Z}_7 \times \mathbf{Z}_7$

(D) cyclic

23. In the ring $\mathbf{Z}[i]$ of Gauss integers

(A) 3 and 5 are irreducible

(B) 2 and 3 are not irreducible

(C) 2 and 5 are irreducible

(D) 3 is irreducible but 2 is not

24. Consider field extensions $Q(\sqrt{2})$ and $Q(\sqrt[3]{5})$ of Q. Then

(A) $Q(\sqrt{2})$ and $Q(\sqrt[3]{5})$ are Galois extension of Q

(B) $Q(\sqrt{2})$ is a Galois extension but $Q(\sqrt[3]{5})$ is not

(C) none of them are Galois extension

(D) $Q(\sqrt[3]{5})$ is a Galois extension but $Q(\sqrt{2})$ is not

25. The sum of divisors of 200 is

(A) 460 (B) 465

(C) 475 (D) 480

26. The property that any linear combination of solutions of an ordinary differential equation is again a solution of that differential equation, holds for

(A) all linear differential equations

(B) all exact differential equations

(C) all homogeneous linear differential equations

(D) all first order non-linear differential equations

27. The initial value problem $\frac{dy}{dx} = y^{1/3}, y = 0$ has

(A) a unique solution

(B) two solutions, one of which is a trivial solution

(C) two non-trivial solutions

(D) three solutions

28. The complete integral of $f(p, q) = 0$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ is

(A) $ax + by + cz^2 = 0$, with $f(a, b) = 0$

(B) $ax^2 + by^2 + cz^2 = 0$, with $f(a, b) = 0$

(C) $z = ax + by$, with $f(a, b) = 0$

(D) $z = \frac{x}{a} + \frac{y}{b}$, with $f(a, b) = 0$

29. If the geometrical equations of a dynamical system do not contain the time t explicitly, then

(A) the Lagrangian is equal to the sum of the kinetic and potential energies

(B) Lagrangian is equal to twice the kinetic energy

(C) the sum of kinetic and potential energies is constant

(D) the difference of the kinetic and potential energies is constant

30. Which of the following is *not* true ?

(A) $u(x, t) = x^2 + 9t^2$ is a solution of one-dimensional heat equation

(B) $u(x, t) = e^{-4t} \cos 3x$ is a solution of one-dimensional heat equation

(C) $u(x, t) = \sin t \sin 4x$ is a solution of one-dimensional wave equation

(D) $u(x, t) = \sin 2x \cos 3t$ is a solution of one-dimensional wave equation

31. The necessary condition that $y = \phi(x)$ minimizes the integral $\int_a^b F(x, y, y')\,dx$ is

(A) $\frac{d}{dx}F_y(x, \phi, \phi') - F_{y'}(x, \phi, \phi') = 0$

(B) $\frac{d}{dx}F_{y'}(x, \phi, \phi') - F_y(x, \phi, \phi') = 0$

(C) $\frac{d^2}{dx^2}F_{y'}(x, \phi, \phi') - F_y(x, \phi, \phi') = 0$

(D) $\frac{\partial}{\partial x}F_{y'}(x, \phi, \phi') - F_x(x, \phi, \phi') = 0$

32. If the kinetic and potential energies for a particle are given by $T = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2)$ and $V = -\frac{\mu}{r}$, where (r, θ) are its polar coordinates, then the Hamiltonian for the system is :

(A) $H(r, \theta; p_r, p_\theta) = \frac{1}{2}\left(p_r^2 - p_\theta^2\right) + \frac{\mu}{r}$

(B) $H(r, \theta; p_r, p_\theta) = \frac{1}{2}\left(p_r^2 - \frac{p_\theta^2}{r^2}\right) + \frac{\mu}{r}$

(C) $H(r, \theta; p_r, p_\theta) = \frac{1}{2}\left(p_r^2 + \frac{p_\theta^2}{r^2}\right) - \frac{\mu}{r}$

(D) $H(r, \theta; p_r, p_\theta) = \frac{1}{2}\left(p_\theta^2 - \frac{p_r^2}{\theta^2}\right) - \frac{\mu}{r}$

33. If we apply Euler-Cauchy method to solve the initial value problem $\frac{dy}{dx} = x + y,\ y(0) = 0$, taking $h = 0.2$, then we get

(A) $y_{n+1} = y_n + 0.2(x_n + y_n)$

(B) $y_{n+1} = y_n + 0.2(x_{n-1} + y_{n-1})$

(C) $y_{n+1} = y_{n-1} + 0.2(x_n + y_n)$

(D) $y_{n+1} = y_{n-1} + (.2)^n(x_n + y_n)$

34. General solution of $x^2\frac{d^2y}{dx^2} - \frac{5}{2}x\frac{dy}{dx} - 2y = 0$ is

(A) $y = c_1x^{1/2} + c_2x^4$

(B) $y = c_1x^{-1/2} + c_2x^4$

(C) $y = c_1x^{3/2} + c_2x^2$

(D) $y = c_1x^{-1/2} + c_2x^2$

35. The curve joining the points (x_1, y_1) and (x_2, y_2) so that the surface of revolution given by $\int_{x_1}^{x_2} 2\pi y\sqrt{1 + (y')^2}\, dy$ is minimum is

(A) $y = c_1e^x + c_2e^{-x}$

(B) $y = c_1 \sinh\left(\frac{x - c_2}{c_1}\right)$

(C) $x = c_1 \log\left(\frac{y + \sqrt{y^2 - c_1^2}}{c_1}\right) + c_2$

(D) $y = c_1 \log\left(\frac{x + \sqrt{x^2 - c_1^2}}{c_1}\right) + c_2$

36. In the motion of a rigid body about one of its points, which is kept fixed, if the moment of the external forces about the fixed point is steadily zero, then its

(A) only angular momentum is conserved

(B) angular momentum and kinetic energy both are conserved

(C) only kinetic energy is conserved

(D) angular velocity is a constant

37. The two-dimensional harmonic equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ is

(A) hyperbolic and is in canonical form

(B) hyperbolic but not elliptic

(C) parabolic and is in canonical form

(D) elliptic and is in canonical form

38. For a r.v. X with EX = θ and V(X) = $\sigma^2 < \infty$ Chebyshev's inequality is obtained to be $P[|X - \theta| \leq 8] \geq \frac{13}{16}$. Then which of the following is *true* ?

(A) $\sigma^2 = 4$ (B) $\sigma^2 = 8$

(C) $\sigma^2 = 12$ (D) $\sigma^2 = 16$

39. The characteristic function of a r.v. X is given by $\phi_X(t) = \left(\frac{1}{3} + \frac{2}{3}e^{it}\right)^4$. Then which of the following is *true* about the first two moments of X ?

(A) $EX = \frac{4}{3}$ and $EX^2 = \frac{8}{3}$ (B) $EX = \frac{4}{3}$ and $EX^2 = \frac{4}{3}$

(C) $EX = \frac{8}{3}$ and $EX^2 = 8$ (D) $EX = \frac{8}{3}$ and $EX^2 = \frac{8}{9}$

40. Suppose state i leads to state j. Then which of the following is *true* ?

(A) $p_{ij}^{(n)} > 0$ for all $n \geq 1$ (B) $p_{ij}^{(1)} > 0$

(C) $p_{ij}^{(n)} > 0$ for some $n \geq 1$ (D) $p_{ij}^{(n)} > 0$ for all n large

41. Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. N(0, 1) r.v.s. Let $U = \max(X_1, \ldots, X_n)$ and $V = \min(X_1, \ldots, X_n)$ which of the following is *true* ?

(A) U and V are identically distributed

(B) U and V are independent

(C) U and –V are identically distributed

(D) U and –V are independent

42. Let X be a standard normal r.v. with p.d.f. f and Y be a normal r.v. with mean zero and variance 9. If g is the p.d.f. of Y, which of the following is *true* ?

(A) f and g do not intersect

(B) f and g intersect at O

(C) f and g intersect at two points

(D) f and g intersect at more than two points

43. Let $x_1, x_2, \ldots x_n$ be a random sample from N(0, σ^2), then UMP test for testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 > \sigma_0^2$ is based on :

(A) $\sqrt{\Sigma x_i^2}$ (B) $\Sigma(x_i - \overline{x})^2$

(C) $\Sigma|x_i - \overline{x}|$ (D) Σx_i^2

44. Let U and L denote respectively the lower and upper limits of the 95% C.I. for the mean μ of a normal distribution. Which of the following statements is *false* ?

(A) The probability that interval (L, U) contains population mean μ is 0.95

(B) The larger the sample size, the samller the distance U – L

(C) The sample mean $\overline{x}$ is midway between L and U

(D) The probability that population mean μ is the mid-point of L and U is 0.05.

45. For absolute error loss function which of the following is the Bayes estimator ?

(A) Mean of the posterior distribution

(B) Mode of the posterior distribution

(C) Median of the posterior distribution

(D) Range of the posterior distribution

46. For a linear model

$$E(y_1) = \theta_1 + \theta_2 + \theta_3,$$

$$E(y_2) = \theta_1 + \theta_3,$$

$$E(y_3) = \theta_2.$$

Which of the following conditions is *correct* for estimability of $\underline{\lambda}' \underline{\theta} = \lambda_1\theta_1 + \lambda_2\theta_2 + \lambda_3\theta_3$?

(A) $\lambda_1 = \lambda_2$

(B) $\lambda_1 = \lambda_3$

(C) $\lambda_2 = \lambda_3$

(D) $\lambda_1 = \lambda_2 + \lambda_3$

47. If $X_{(r)}$ denotes failure time of rth item (r being prefixed), then for failure-censored sample (FCS) and time-censored sample (TCS). Which one of the following is *true* ?

(A) $X_{(r)}$ is a constant in FCS and TCS

(B) $X_{(r)}$ is a random variable in FCS and TCS

(C) $X_{(r)}$ is a constant in FCS and a random variable in TCS

(D) $X_{(r)}$ is a random variable in FCS and a constant in TCS

48. A random sample of size n is drawn from a bivariate normal population to test if the population correlation coefficient ρ is zero. Let r denote the sample correlation coefficient. Which of the following is the appropriate test statistic with the stated distribution when $\rho = 0$?

(A) $\frac{(n-2)r}{1-r^2}$ which follows student's t distribution with $(n - 2)$ d.f.

(B) $\frac{\sqrt{n-2}\,r}{\sqrt{1-r^2}}$ which follows student's t distribution with $(n - 2)$ d.f.

(C) $\frac{\sqrt{(n-1)}\,r}{\sqrt{1-r^2}}$ which follows student's t distribution with $(n - 1)$ d.f.

(D) $\frac{(n-1)r}{(1-r^2)}$ which follows student's t distribution with $(n - 1)$ d.f.

49. If in a BIBD, $v = b$, then which of the following is *not* true ?

(A) $bk = vr$

(B) $\lambda(v-1) = r(k-1)$

(C) $r - \lambda$ is a perfect square for even v

(D) $\frac{k^2}{v}$ is an integer

50. If $\underline{c}'\underline{x}$ and $\underline{b}'\underline{y}$ are primal maximization and dual minimization objective functions, which one of the following is *true* in general ?

(A) $\underline{c}'\underline{x} = \underline{b}'\underline{y}$

(B) $\underline{c}'\underline{x} \geq \underline{b}'\underline{y}$

(C) $\underline{c}'\underline{x} \leq \underline{b}'\underline{y}$

(D) No specific condition can be drawn

ROUGH WORK