MATHEMATICAL SCIENCE

PAPER - III

Signature of Invigilators

1. ....

2. ....

Roll No. ☐☐☐☐☐

(In figures as in Admit Card)

Roll No. ....

....

(in words)

JY–06/01

Name of the Areas/Section (if any)...............

Time Allowed : 2-1/2 hours] **[Maximum Marks : 200**

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page.
2. Write name of your Elective/Section if any,
3. Answer to short answer/essay type questions are to be written in the space provided below each question or after the questions in test booklet itself. No additional sheets are to be used.
4. Read instructions given inside carefully.
5. Last page is attached at the end of the test booklet for rough work.
6. If you write your name or put any special mark on any part of the test booklet which may disclose in any way your identity, you will render yourself liable to disqualification.
7. Use of calculator or any other Electronics Devices is prohibited.
8. There is no negative marking.
9. You should return the test booklet to the invigilator at the end of the examination and should not carry any paper outside the examination hall.

પરીક્ષાર્થીઓ માટે સૂચનાઓ :

૧. આ પૃષ્ઠના ઉપલા ભાગે આપેલી જગ્યામાં તમારી ક્રમાંક સંખ્યા (રોલ નંબર) લખો.
૨. તમે જે વિકલ્પનો ઉત્તર આપો તેનો સ્પષ્ટ નિર્દેશ કરો.
૩. ટૂંકનોંધ કે નિબંધ પ્રકારના પ્રશ્નોના ઉત્તર દરેક પ્રશ્નની નીચે આપેલી જગ્યામાં જ લખો. વધારાના કોઈ કાગળનો ઉપયોગ કરશો નહી.
૪. અંદર આપેલી સૂચનાઓ ધ્યાનથી વાંચો.
૫. આ ઉત્તરપોથીમાં અંતે આપેલું પૃષ્ઠ કાચા કામ માટે છે.
૬. આ ઉત્તરપોથીમાં ક્યાંય પણ તમારી ઓળખ કરાવી દે એવી રીતે તમારું નામ કે કોઈ ચોક્કસ નિશાની કરી હશે તો તમને આ પરીક્ષા માટે ગેરલાયક ગણવામાં આવશે.
૭. કેલક્યુલેટર અથવા ઈલેક્ટ્રોનિક્સ સાધનો નો ઉપયોગ કરવો નહી.
૮. નકારાત્મક ગુણાંકપદ્ધતિ નથી.
૯. પ્રશ્નપત્ર લખાઈ રહે એટલે આ ઉત્તરપોથી તમારા નિરીક્ષકને આપી દેવી. પરીક્ષાખંડની બહાર કોઈ પણ પ્રશ્નપત્ર લઈ જવું નહીં.

FOR OFFICE USE ONLY
MARKS OBTAINED

Question Number	Marks Obtained	Question Number	Marks Obtained	Question Number	Marks Obtained
1.		18.		35.	
2.		19.		36.	
3.		20.		37.	
4.		21.		38.	
5.		22.		39.	
6.		23.		40.	
7.		24.			
8.		25.			
9.		26.			
10.		27.			
11.		28.			
12.		29.			
13.		30.			
14.		31.			
15.		32.			
16.		33.			
17.		34.			

Total Marks obtained ....

Signature of the co-ordinator ....
(Evaluation)

SEAL

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

No.	log		No.	log
$\pi = 3.14159$	0.49715	$\ln x = \log_e x = (1/M) = \log_{10} x$	$(1/M) = 2.30259$	0.36222
$e = 2.71828$	0.43429	$\log x = \log_{10} x = M \log_e x$	$M = 0.43429$	$\bar{1}.63778$

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

p	1	2	3	4	5	6	7	8	9	10
$\log e^{p}$	0.4343	0.8686	1.3029	1.7372	2.1715	2.6058	3.0401	3.4744	3.9087	4.3429
$\log e^{-p}$	$\bar{1}.5657$	$\bar{1}.1314$	$\bar{2}.6971$	$\bar{2}.2628$	$\bar{3}.8285$	$\bar{3}.3942$	$\bar{4}.9599$	$\bar{4}.5256$	$\bar{4}.0913$	$\bar{5}.6571$

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences								
											1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

MATHEMATICAL SCIENCE
Paper - III

NOTES :

(a) This paper contains **FORTY (40)** questions, each carrying **twenty (20)** marks. The first **twenty (20)** pertain to mathematics, the remaining to statistics.

(b) Attempt **any ten (10)** questions.

(c) Answer each question in not more than **300 words.**

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1. (a) Let f (a, b) IR, be a nonnegative valued continuous function. Show that,

$$f(x) = 0, \ \forall x \in [a, b] \quad \text{if} \quad \int_a^b f(x)dx = 0.$$

(b) Let $E = [-1, 1] \times [-1, 1]$, be the rectangle in plane.

If $f : E \to IR$ is continuous as a function of two variables, show that f is continuous in each variable separately. Show that, the converse is false, by exhibiting an example.

2. Suppose $\sum_{n=0}^{\infty} a_n$ is a series of complex numbers which converges. Define,

$$f(z) = \sum_{n=0}^{\infty} a_n z^n \quad , \quad |z| < 1.$$ Show that

f(z) tends to $\sum_{n=0}^{\infty} a_n$, as z approaches 1 in such a way that

$$\frac{|1-z|}{1-|z|}, \quad |z| < 1$$ remains bounded.

3. Give an example of a unique factorization domain. Obtain two distinct factorizations of 6, 9 and 21 in

$$\mathbf{Z} = \left(\sqrt{-5}\right) = \{a + b\sqrt{-5} : a, b \in \mathbf{Z}\}.$$

4. (a) Let g be integrable over a measurable set E and let $< f_n >$ be a sequence of measurable functions such that $|f_n| \le g$ on E and $f(x) = \lim_{n\to\infty} f_n(x)$ for almost all x in E. Show that $\int_E f = \lim_{n\to\infty} \int_E f_n$.

(b) Show that if f is integrable over a measurable set E, then $|f|$ is integrable over E. Also show that $\left|\int_E f\right| \le \int_E |f|$.

5. If a finite field F contains p^n elements then show that each element x of F satisfies the polynomial $x^{p^n} - x$.

Are there finite fields with (a) 121 (b) 72 (c) 343 and (d) 250 elements.

6. Prove that a normed linear space X is finite dimensional iff every linear functional on X is continuous.

7. (a) Show that any connected metric space with atleast two elements, must be uncountable.

(b) Give two topological spaces X and Y such that X is homeomorphic to a subspace of Y, Y is homeomorphic to a subspace of X but X and Y are not homeomorphic.

8. Show that there are at least two vertices of degree 1 in a tree with at least two vertices.

9. (a) Using the method of successive approximation, obtain solution of the initial value problem

$$\frac{dy}{dx} = xy \quad ; \quad y(o) = 1.$$

(b) Discuss the method of solving linear partial differential equation of first order

$$Pp + Qq = R$$

where P, Q and R are functions of x, y, z and $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

10. For every positive integer n show that $\sum_{d|n} \phi(d) = n$ where ϕ is the Euler function.

11. A top is spining about its symmetric axis. If one point of the symmetric axis is fixed, then find equations of motion. Discuss geometrically the motion of the top.

12. (a) Define strain energy function. Discuss relation between strain and stress tensor of a fluid in motion.

(b) What is Hook's law? Discuss application of the Hook's law.

13. (a) Prove that the equation of continuing of a fluid flow is $\frac{\partial \rho}{\partial t} + \Delta(\rho \bar{q}) = 0$, where ρ and $\bar{q}$ are respectively density and velocity. Show also that in case of an incompressible fluid, the equation of continuity reduces to the form $\nabla \cdot \bar{q} = 0$.

(b) Define complex potential and complex velocity of a fluid flow. State circle theorem. Using the circle theorem find complex potential of a fluid flow near a circluar cylinder placed in an uniform stream whose velocity at infinity is U along negative x-axis.

14. Define a surface in E^3. Show that the sphere

$$S^2 = \{(x_1, x_2, x_3) \in E^3 \mid x_1^2 + x_2^2 + x_3^2 = 1\}$$ is a surface in E^3.

15. Discuss techniques of the calculus of variation and using it and Hamilton's principle, obtain Lagranges equations of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i} = 0, \; i=1,2,\ldots,n.$$

16. Let f(t) and f'(t) be continuous for $t \geq 0$ and of exponetial order. Then prove that

$$L[f'(t)] = sF(s) - f(0),$$

where $F(s) = L[f(t)]$.

17. (a) The iteration method for finding the roots of the equation $f(x) = 0$ required to write the equation in the form $x = \phi(x)$. Starting with x_o as approximate value of the desired root, one gets successive approximation $x_1 = \phi(x_o)$, $x_2 = \phi(x_1)$, ... $x_n = \phi(x_{n-1})$. In what way one should choose ϕ in order that the sequence $x_o, x_1, \ldots x_n$ conveges to the root ? Show geometrically the cases of convergence and divergence of the sequence $x_o, x_1, x_2, \ldots x_n$.

(b) To find a real reoot between 0 and 1 of the equation $f(x) = x^3 + x^2 - 1 = 0$ by the method of iteration, rewrite the equation in the form $x = \phi(x)$ such that the successive approximation sequence converges to the root.

18. Show that $F\left(\frac{1}{1+t^2}\right) = \frac{1}{2}e^{-|\omega|}$.

19. Let H be a separable Hilbert space and $T \in \mathcal{B}(H)$ be a normal operator. Show that $\|T^2\| = \|T\|^2$ and then derive that spectral radius, r (T), of T is equal to $\|T\|$.

20. (a) Find solution of the initial value problem $\frac{dy}{dx} = 3y + 1; y(0) = 2$.

(b) If $\alpha_r D + B_r D' + \gamma_r$ is a factor of F(D, D') and $\phi_r(\xi)$ is an arbitrary function of single variable ξ, $\alpha_r \neq 0$ then prove that

$$y_r = \exp\left(-\frac{\gamma_r x}{\alpha_r}\right)\phi_r(\beta_r x - \alpha_r y)$$

is a solution of the equation F(D, D') z = 0.

21. Maximize $Z = 5x_1 + 4x_2$

Subject to $x_1 + x_2 \leq 5$

$10x_1 + 6x_2 \leq 45$, $x_1, x_2 \geq 0$, integers.

22. Let $\{A_n\}$ be a sequence of events in a probability sapce $(\Omega, \mathcal{A}, P)$. If $A_n \subset A_{n+1}$ for every $n \geq 1$ and if $A = \bigcup_{n=1}^{\infty} A_n$ then show that $\lim_{n \to \infty} P(A_n) = P(A)$

If $\{A_n\}$ is any sequence of events in $(\Omega, \mathcal{A}, P)$ and if $B_n = \bigcap_{k=n}^{\infty} A_k$ for $n \geq 1$ then find $\lim_{n \to \infty} P(B_n)$

23. State and prove Levy-Lindeberg version of Central limit theorem. Hence show that if Z_n is chi-quare with n degrees of freedom then $P(Z_n \leq n) \to \frac{1}{2}$ as $n \to \infty$.

24. State Holder's inequality and using this inequality prove Minkowski's inequality.

25. Define the characteristic function of a distribution function F and show that it always exists.

Obtain the characteristic function of a normal distribution with mean μ and variance σ^2. Hence or otherwise find E[cos X] where X ~ (μ, σ^2).

26. For a random variable X let $\beta_r = E|X|^r$

Prove that $\beta_r^{1/r} \leq \beta_{r+1}^{1/(r+1)}$ for $r = 1, 2, 3, \ldots\ldots$

State precisely any result you wish to use.

27. Let $X_1, X_2, \ldots.. X_n$ be i.i.d Poisson P(λ) r.v.s.

Let $\overline{X} = \sum_{i=1}^{n} \frac{X_i}{n}$, $S^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (X_i - \overline{X})^2$.

Show that $\alpha\overline{X} + (1-\alpha)S^2$ is unbiased for λ for any α real. Find the value of α for which the estimator is UMVU.

28. Let x_1, x_2, ... x_n be a random sample from a uniform distribution U(0, θ)

(i) Find M.L.E. of θ

(ii) Find an unbiased estimator of θ which is a function of M.L.E. obtained in (i).

(iii) Obtain the variance of the unbiased estimator obtained in (ii).

29. In a multinomial model the cell probabilities and observed frequencies are known to be

Prob.	$\theta/2$	$(1 - \theta)/3$	$(1 + \theta)/3$	$(2-3\theta)/6$
Frequency	6	20	28	18

Obtain M L E of θ.

30. Let $X_1, \ldots, X_n$ be a random sample from the distribution with p.d.f.

$$f(x, \mu, \sigma) = \frac{1}{\sigma} \exp\{-(x-\mu)/\sigma\}$$

$$x > \mu, \ \mu \in R, \sigma > 0.$$

Let $X_{(1)}, X_{(2)}, \ldots X_{(n)}$ be order statistics and $D_i = X_{(i)} - X_{(i-1)}$, $i = 1, 2, \ldots n$ and $X_{(0)} = 0$.

Obtain the joint distribution of $D_1, D_2, \ldots D_n$. Hence or otherwise show that for $1 \le r \le s < k \le n$, $X_{(r)}$ and $X_{(k)} - X_{(s)}$ are independent.

31. Let $X_1, X_2, \ldots$ be i.i.d $U(\theta - 1, \theta + 1)$ r. v.s. Suggest three consistent estimtors T_1, T_2 and T_3 respectively based on minimum, median and maximum of a random sample of size 2n+1. Carryout their performances based on mean squared errors.

32. Let $\underline{X} \sim N_p(\mu, \Sigma)$, $\underline{X} = \begin{pmatrix} \underline{X}_1 \\ \underline{X}_2 \end{pmatrix}$, $\mu = \begin{pmatrix} \underline{\mu}_1 \\ \underline{\mu}_2 \end{pmatrix}$, $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$

(i) Prove that $\underline{X}_1$ and $\underline{X}_2$ are independent if and only if $\Sigma_{12} = 0$.

(ii) Obtain conditional distribution of $\underline{X}_2$ given $\underline{X}_1$.

33. State Gauss-Markov Theorem. Define estimability of a linear parametric function.

Let Y_i (i = 1, 2, 3) be independent observations with common variance and

$$E\,Y_1 = \theta_1 - \theta_2$$
$$E\,Y_2 = \theta_1 + \theta_2 - \theta_3$$
$$E\,Y_3 = \theta_1 - \theta_3$$

Examine for the estimability of every linear combination of the parameters. Obtain BLUE of $\theta_1 - \theta_2$.

34. (a) Define ratio estimator $\hat{Y}_R$ of population total and obtain $V(\hat{Y}_R)$.

(b) In large samples with simple random sampling obtain the condition under which estimator $\hat{Y}_R$ has a smaller variance than the estimator $\hat{Y} = N\overline{Y}$.

35. Define Youden square design. Give a complete analysis of a Youden square design.

36. Define (i) moving average process (ii) ARMA (p, q) process.

Explain how ARMA (p, q) can be expressed as an infinite moving average process.

37. Suppose $\{X_n\}$ is an irreducible Markov chain with state spae I = {1, 2,....., M}. Prove that there exists a unique stationary distribution $\pi = \{\pi_1, ...\pi_M\}$ for $\{X_n\}$. Suppose the distribution of X_0 is π. What is the distribution of X_r ?

38. (a) Describe basic concepts essential to construct a life table.

(b) State the assumptions on which the construction of life table is based.

39. Consider a single period inventory model for an item with a random demand. The purchase cost of the item is c, unit holding cost is h and shortage cost is p. If there is no ordering cost and the aim is to minimize the expected total cost, develop the model and derive the optimal order quantity.

40. A 4 ton vessel is loaded with one or more of three items. The following table gives the unit weight, w_i, in tons and the unit revenue r_i, in lacs of rupees for item i. Discuss how the vessel should be loaded to maximize the total return by dynamic programming.

Item (i)	w_i	r_i
1	3	47
2	2	31
3	1	14

Note : Fractional item can not be loaded.