Signature of Invigilators

1. ....

2. ....

MATHEMATICAL SCIENCES

Paper II

Roll No. | | | | | | (In figures as in Admit Card)

Roll No. ....

....

(In words)

JY—04/1

Name of the Areas/Section (if any) ....

Time Allowed : 75 Minutes] [Maximum Marks : 100

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page.
2. This paper consists of *seventy four* **(74)** multiple choice type questions. Attempt any *fifty* **(50)** questions.
3. Each item has upto four alternative responses marked (A), (B), (C) and (D). The answer should be a capital letter for the selected option. The answer letter should entirely be contained within the corresponding square.

   Correct method **A** Wrong Method A or **A**
4. Your responses to the items for this paper are to be indicated on the ICR Answer Sheet under paper II only
5. Read instructions given inside carefully.
6. One sheet is attached at the end of the booklet for rough work.
7. You should return the test booklet to the invigilator at the end of paper and should not carry any paper with you outside the examination hall.

પરીક્ષાર્થીઓ માટેની સૂચનાઓ :

૧. આ પાનાની ટોચમાં દર્શાવેલી જગ્યામાં તમારો રોલ નંબર લખો.

૨. આ પ્રશ્નપત્રમાં **ચવોહત્તેર (74)** બહુવૈકલ્પિક ઉત્તરોવાળા પ્રશ્નો છે. કોઈપણ **(50)** પ્રશ્નોના જવાબ આપો.

૩. પ્રત્યેક પ્રશ્ન વધુમાં વધુ ચાર બહુવૈકલ્પિક ઉત્તરો ધરાવે છે. જે (A), (B), (C) અને (D) વડે દર્શાવવામાં આવ્યા છે. પ્રશ્નનો ઉત્તર કેપીટલ સંજ્ઞા વડે આપવાનો રહેશે. ઉત્તરની સંજ્ઞા આપેલ ખાનામાં બરાબર સમાઈ જાય તે રીતે લખવાની રહેશે.

   ખરી રીત : **A** ખોટી રીત : A , **A**

૪. આ પ્રશ્નપત્રના જવાબ આપેલ ICR Answer Sheetના Paper II વિભાગની નીચે આપેલ ખાનાંઓમાં આપવાના રહેશે.

૫. અંદર આપેલ સૂચનાઓ કાળજીપૂર્વક વાંચો.

૬. આ બુકલેટની પાછળ આપેલું પાનું રફ કામ માટે છે.

૭. પરીક્ષાસમય પૂરો થઈ ગયા પછી આ બુકલેટ જે તે નિરીક્ષકને સોંપી દેવી. કોઈ પણ કાગળ પરીક્ષાખંડની બહાર લઈ જવો નહીં.

MATHEMATICAL SCIENCES

PAPER-II

Note : This paper contains **seventy four (74)** multiple-choice questions. Attempt any fifty (50) questions only each question carries **two (2)** marks.

1. With the help of the Rolle's theorem, it can be seen that the equation :

$$4x^3 - 18x^2 + 22x - 6 = 0 \text{ has}$$

(A) one root in each of the intervals (0, 1), (1, 2) and (2, 3)

(B) one root in each of the intervals (–1, 0), (1, 2) and (2, 3)

(C) two roots in (0, 1) and one root in (1, 2)

(D) no root in (0, 1) and no root in (1, 2)

2. Let $u_n = n^2 x^n$. Then $\lim_{n\to\infty} u_n =$.........

(A) 0 if $x > 0$ (B) 1 if $x < 1$

(C) 1 if $x > 1$ (D) ∞ if $x > 1$

3. Let $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0, \end{cases}$

$$g(x) = \begin{cases} x^2\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Then :

(A) f and g are differentiable at 0

(B) f is differentiable at 0 but g is not differentiable at 0

(C) g is differentiable at 0 but f is not differentiable at 0

(D) neither f nor g is differentiable at 0

4. The polar form of the complex number $-\sqrt{3} + i$ is :

(A) $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ (B) $2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(C) $2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ (D) $2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

5. The power series $4z + 3z^2 - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \ldots$ represents which of the following functions ?

(A) $\sin z + \frac{5}{2}z^2$

(B) $\sin z + 3z + 3z^2$

(C) $\cos z + 4z + \frac{7}{2}z^2$

(D) $\sin z + 3z + 2z^2$

6. Let $f(z) = \frac{(z+2)\tan z}{z^2}$.

Then $f(z)$ has a pole at 0 of order a and residue b, where :

(A) $a = 2, b = 0$

(B) $a = 1, b = 2$

(C) $a = 2, b = 1$

(D) $a = 2, b = 2$

7. If A, B are 3 × 3 matrices such that det A = 3 and det B = 2, then :

(A) AB^{-1} may not be defined

(B) AB^{-1} is defined and det $(AB^{-1}) = 6$

(C) AB^{-1} is defined and det $(AB^{-1}) = 5$

(D) AB^{-1} is defined and det $(AB^{-1}) = \frac{3}{2}$

8. If A is $m \times n$ and B is $n \times p$ matrices with ranks r_A and r_B respectively and rank of (AB) = r, then which one of the following is always true ?

(A) $r = \min\{r_A, r_B\}$

(B) $r = \max\{r_A, r_B\}$

(C) $r \leq \min\{r_A, r_B\}$

(D) $r \geq \max\{r_A, r_B\}$

9. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be defined by $T(x, y) = (x + y, 3x - 4y)$. Then :

(A) T is not linear

(B) T is linear, but T^{-1} does not exist

(C) T is linear, T^{-1} exists and the matrix for T^{-1} is $\begin{bmatrix} 4/7 & 1/7 \\ 3/7 & -1/7 \end{bmatrix}$

(D) T is linear, T^{-1} exists and the matrix for T^{-1} is $\begin{bmatrix} 4/7 & -1/7 \\ -3/7 & 1/7 \end{bmatrix}$

10. If $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$, then eigenvalues of $2A^2 - A + 3$ are :

(A) 3, 2, 5

(B) 18, 9, 48

(C) 18, 8, 50

(D) 15, 6, 45

11. If the quadratic form $x^2 + 2y^2 + 6z^2 - 2xy + 4zx$ is expressed as XAX', where

$$X = (x,\ y,\ z),\ A = \begin{pmatrix} -1 & p & 2 \\ -1 & 2 & 0 \\ 2 & 0 & q \end{pmatrix},\ \text{then :}$$

(A) $p = 1, q = 2$ (B) $p = -1, q = 6$

(C) $p = 0, q = 4$ (D) $p = -1, q = 4$

12. Let T be a linear transformation from a finite dimensional vector space V to V. Consider the following statements :

p : T is one-to-one on V.

q : T is invertible.

r : T maps any linearly independent subset of V onto a linearly independent subset.

s : 0 is not an eigenvalue of T.

Then :

(A) p is equivalent to q, but not equivalent to r

(B) p is equivalent to r, but not equivalent to s

(C) p is equivalent to s, but not equivalent to q

(D) p, q, r, s are all equivalent to one another

13. Suppose a coin is tossed till a head turns up. Let X be the total number of tosses made. If p is the probability that a head turns up in a single toss, the distribution of X is given by

$$P_k = P(X = K),$$

where :

(A) $P_k = (1 - p)p^k,\ k = 0, 1, 2, \ldots$

(B) $P_k = p(1 - p)^k,\ k = 0, 1, 2, \ldots$

(C) $P_k = p(1 - p)^{k-1},\ k = 1, 2, \ldots$

(D) $P_k = (1 - p)p^{k-1},\ k = 1, 2, \ldots$

14. Let X be the binomial r.v. with expectation = 8 and variance = 4. Then :

(A) $P(X = 0) = 8\ P(X = 1)$

(B) $P(X = 1) = 8\ P(X = 0)$

(C) $P(X = 1) = 16\ P(X = 0)$

(D) $P(X = 0) = 16\ P(X = 1)$

15. The probability density function of a random variable is given by

$$f(x)=\begin{cases} c/x^2 & , \text{ if } |x|>1 \\ 0 & , \text{ if } |x|\le 1, \end{cases}$$

where c is a constant. Then :

(A) $c = 1$ (B) $c = 2$

(C) $c = \frac{1}{2}$ (D) $c = \frac{1}{4}$

16. Suppose X is a random variable taking values +1 and –1 only with probabilities $c/3$ and $c/6$ respectively. Let $Y = X^2$. Then :

(A) $c = 1$ and $P(Y = 0) = 1$ (B) $c = 1$ and $P(Y = 1) = 1$

(C) $c = 2$ and $P(Y = 0) = 1$ (D) $c = 2$ and $P(Y = 1) = 1$

17. An unbiased coin is tossed twice. Let X and Y denote the number of times a head turns up and the number of times a tail turns up respectively. Then which of the following is FALSE ?

(A) $P(X > Y) = P(X < Y)$ (B) $P(X + Y = 2) = 1$

(C) $P(X = 0) = P(Y = 0)$ (D) $P(X = Y) = \frac{1}{2}$

18. An unbiased coin is tossed once. If head turned up, then the coin is tossed once more. If, on the other hand, the first toss resulted in a tail, the coin is tossed twice more. Then the probability of getting tail two times in all is :

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$

(C) $\frac{3}{8}$ (D) $\frac{1}{2}$

19. Consider an LPP in n variables with m ($< n$) constraints and non-negative restrictions on all variables. The set of feasible solutions is a convex set bounded by :

(A) n hyperplanes

(B) m hyperplanes

(C) $m + n$ hyperplanes each of which is n dimensional

(D) $m + n$ hyperplanes each of which is m dimensional

20. Consider the LPP :

Min. : $Z = 3x + 6y$

Subject to :

$x + y \geq 5$

$2x + y \geq 9$

$x, y \geq 0.$

Then an optimal solution is :

(A) $\left(\frac{9}{2}, 0\right)$ (B) (4, 1)

(C) (5, 0) (D) (0, 5)

21. If E is infinite and countable and $E = \bigcup_{i=1}^{\infty} E_i$, then :

(A) at least one E_i is infinite

(B) infinite number of E_is are infinite

(C) no E_i need be infinite

(D) if E_is are pairwise disjoint at least one E_i is infinite

22. Let f and g be real valued uniformly continuous functions on **R**. Then :

(A) fg is uniformly continuous on **R**

(B) fg may not be uniformly continuous, but if one of f, g is bounded, then fg is uniformly continuous

(C) if both f and g are bounded, then fg is uniformly continuous

(D) fg may not be uniformly continuous even if f and g are bounded

23. Let f be a real valued continuous function on **R**. Let (x_n) be a sequence of real numbers. Which of the following is *false* ?

(A) (x_n) is convergent $\Rightarrow (f(x_n))$ is convergent

(B) (x_n) is Cauchy $\Rightarrow (f(x_n))$ is Cauchy

(C) (x_n) is bounded $\Rightarrow (f(x_n))$ is bounded

(D) (x_n) is not convergent $\Rightarrow (f(x_n))$ is not convergent

24. Let X be a non-empty set and d and ρ be metrics on X. Then :

(A) every $d + \rho$-open subset of X is both d-open and ρ-open

(B) every d-open or ρ-open subset of X is $d + \rho$ open

(C) No $d + \rho$-open subset of X can be d-open or ρ-open, unless it is ϕ or X.

(D) $d + \rho$ may not be a metric on X

25. Let $f(x) = \frac{x}{e^x}$ $(x \geq 0)$. Then :

(A) f is increasing on $[0, \infty)$

(B) f is decreasing on $[0, \infty)$

(C) f increases on $(0, 1)$ and decreases on $(1, \infty)$

(D) f decreases on $(0, 1)$ and increases on $(1, \infty)$

26. The general equation of a line in the complex plane is (here $\alpha \neq 0$, α complex and n real) :

(A) $\alpha z + n = 0$ (B) $\alpha \bar{z} + n = 0$

(C) $\alpha z + \bar{\alpha} \bar{z} + n = 0$ (D) $z = n$

27. Which of the following is *not true* ?

(A) The set of all translations on the complex plane $\mathbb{C}$ is an abelian group

(B) The set of all rotations on $\mathbb{C}$ is a group but the set of all dilations on $\mathbb{C}$ is not a group

(C) A translation maps a (straight) line to a (straight) line and a circle to circle

(D) A non-singular linear transformation maps a line to a line and a circle to a circle

28. C_1 is the arc of the parabola $y = x^2$ from the point (0, 0) to the point (3, 9) and C_2 is the line segment between the same two points. If $\alpha = \int_{C_1} z dz$ and $\beta = \int_{C_2} z dz$, then :

(A) α does not exist but β exists

(B) $\alpha > \beta$

(C) $\alpha = \beta$

(D) $\alpha < \beta$

29. Which of the following functions is analytic in the unit disc, vanishes at 0 and at all $z = \frac{2}{n}$ for every non-zero integer n ?

(A) 0 (B) $\sin \frac{\pi}{2z}$

(C) $\sin \frac{2\pi}{z}$ (D) $z^7 \sin \frac{2\pi}{z}$

30. Take the group S_3 of permutations on three symbols. It is generated by permutations a and b such that $a^3 = e = b^2$ and $ab = ba^2$. Let H and K be subgroups of S_3 given by $H = \{e, b\}$, $K = \{e, a, a^2\}$. Which of the following is true ?

(A) H is a normal subgroup of S_3 but K is not

(B) K is a normal subgroup of S_3 but H is not

(C) Both H and K are normal subgroups of S_3

(D) Neither H nor K is a normal subgroup of S_3

31. The set of all linear functions $f_{ab}(z) = az + b$, where a, b are complex numbers and $a \neq 0$ form a group under ordinary composition of mappings. Which of the following describes this operation *correctly* ?

(A) $f_{ab}f_{cd} = f_{ab,cd}$

(B) $f_{ab}f_{cd} = f_{ad}$

(C) $f_{ab}f_{cd} = f_{ac,ad+b}$

(D) $f_{ab}f_{cd} = f_{bd,ac+d}$

32. How many distinct (mutually non-isomorphic) subgroups does the cyclic group of order 6 have ?

(A) 2

(B) 3

(C) 4

(D) 6

33. H is a subgroup of G and every coset of H in G is also a subgroup of G. Then :

(A) $H = \{e\}$

(B) G must have order 2

(C) H must have order 2

(D) H = G

34. Let R be a ring and R[x, y] the ring of polynomials in two variables over R. The polynomial ring R[x] is :

(A) not a subring of R[x, y] but it is an ideal of R[x, y]

(B) a subring but not an ideal of R[x, y]

(C) both a subring and an ideal of R[x, y]

(D) neither a subring nor an ideal of R[x, y]

35. Let $V = \{(x, y) \mid x, y \in \mathbf{R}\}$, where $\mathbf{R}$ is the field of real numbers. Define addition and scalar multiplication on V as :

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$$
$$\alpha (x_1, y_1) = (0, \alpha y_1)$$

for all $(x_1, y_1), (x_2, y_2) \in V$ and $\alpha \in \mathbf{R}$

Then :

(A) V is a vector space

(B) V is not a vector space because additive identity does not exist

(C) V is not a vector space because it is not closed under scalar multiplication

(D) V is not a vector space because there does not exist element $I \in \mathbf{R}$ such that $I \,.\, v = v$ for all $v \in V$.

36. Let V be a vector space of 2×2 matrices over $\mathbf{R}$ and let $f : V \to V$ be a linear map defined by $f(X) = XM - MX$, where $M = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$. Then :

(A) ker (f) is of dimension 2 and has a basis $\left\{\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$

(B) ker (f) is of dimension 2 and has a basis $\left\{\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}\right\}$

(C) ker (f) is of dimension 1 and has a basis $\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}$

(D) ker (f) is of dimension 1 and has a basis $\left\{\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}\right\}$

37. Let $f : V \to V'$ be a linear mapping defined by

$$f(x, y, z) = (3x + 2y - 4z, x - 5y + 3z),$$

where the basis of V is $B_1 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ and the basis of V' is $B_2 = \{(1, 3), (2, 5)\}$. Then f is represented by the matrix :

(A) $\begin{bmatrix} -7 & -33 & -13 \\ 4 & 19 & 8 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 5 & 3 \\ -1 & -4 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 2 & -4 \\ 1 & -5 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} -13 & -20 & 26 \\ 8 & 11 & -15 \end{bmatrix}$

38. Let V be a subspace of $\mathbf{R}^3$ generated by $\{\overline{u}, \overline{v}\}$, where $\overline{u} = (1, 2, 1)$ and $\overline{v} = (1, 1, 1)$. Suppose for any two vectors $\overline{v}_1 = (x_1, x_2, x_3)$ and $\overline{v}_2 = (y_1, y_2, y_3) \in \mathbf{R}^3$, the scalar product is defined as $\overline{v}_1 . \overline{v}_2 = x_1 y_1 - x_2 y_2 - x_3 y_3$. Then the orthogonal basis of V with respect to the given basis and the scalar product is given by :

(A) $\left\{(1, 2, 1), \left(\frac{1}{2}, 0, \frac{1}{2}\right)\right\}$ (B) $\left\{(1, 2, 1), \left(\frac{1}{2}, 0, -\frac{1}{2}\right)\right\}$

(C) $\left\{(1, 2, 1), \left(-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)\right\}$ (D) $\{(1, 1, 1), (-1, 1, 0)\}$

39. The initial value problem $y' = y^{1/3}, y(0) = 0$ has :

(A) a unique solution (B) exactly three solutions

(C) exactly two solutions (D) no solution

40. A differential equation of first order :

(A) has only one singular solution

(B) may have many singular solutions

(C) does not have a singular solution

(D) has a singular solution if it is exact.

41. The following is *not* a solution of a homogeneous linear differential equation with constant coefficients :

(A) $e^{\lambda x}$, where λ is a constant (B) $\cos^5 x + \sin^{90} x$

(C) $x^3 e^{3x} \sin 3x$ (D) $x \log x$

42. The differential equation $a(x, y) \frac{\partial u}{\partial x} + b(x, y) \frac{\partial u}{\partial y} = c(x, y, u)$ is :

(A) linear

(B) semi-linear

(C) quasi-linear

(D) non-linear in u and its derivatives

43. The differential equation $\frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = u$ has :

(A) no real characteristic curves

(B) two parameter family of characteristic curves

(C) one parameter family of characteristic curves along which u remains constant

(D) one parameter family of characteristic curves as straight lines along which u varies exponentially

44. The general solution of the wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$ involves :

(A) one arbitrary constant

(B) two arbitrary constants

(C) one arbitrary constant and an arbitrary function

(D) two arbitrary functions

45. The coefficient of correlation is invariant under changes of :

(A) location but not scale

(B) scale but not location

(C) both location and scale in both variables

(D) both location and scale but of only one variable

46. Data regarding the number of cars sold in 5 different cities in India during the years 2002 and 2003 are to be represented in a diagramatic form. Which of the following is appropriate for this purpose ?

(A) Pie diagram (B) Histogram

(C) Bar diagram (D) None of these

47. If the joint distribution of two r.v.s X and Y is given by its p.d.f.

$$f(x, y) = \begin{cases} 2 & , 0 < y < x < 1 \\ 0 & , \text{otherwise} \end{cases}$$

the marginal of Y is :

(A) $g(y) = \begin{cases} 2(1-y) & , 0 < y < x \\ 0 & , \text{otherwise} \end{cases}$

(B) $g(y) = \begin{cases} 2 & , 0 < y < x \\ 0 & , \text{otherwise} \end{cases}$

(C) $g(y) = \begin{cases} 2(1-y) & , 0 < y < 1 \\ 0 & , \text{otherwise} \end{cases}$

(D) $g(y) = \begin{cases} 2 & , 0 < y < 1 \\ 0 & , \text{otherwise} \end{cases}$

48. Let $\{X_n\}$ be a sequence of i.i.d. r.v.s with $E|X_1| < \infty$ and Var $(X_1) = \sigma^2 > 0$.

Consider the two statements :

S1 : For $\{X_n\}$ to obey SLLN it is necessary that $\sigma^2 < \infty$.

S2 : For $\{X_n\}$ to obey CLT it is necessary that $\sigma^2 < \infty$.

Which of the following is true ?

(A) Only S1

(B) Only S2

(C) Both S1 and S2

(D) Neither S1 nor S2

49. The joint probability mass function of r.v.s X and Y is given below :

X \ Y	−1	+1
0	1/8	1/8
1	1/4	1/2

Then covariance (X, Y) is :

(A) 0

(B) $\frac{1}{8}$

(C) $\frac{1}{16}$

(D) 1

50. Let A_1, A_2 and A_3 be three independent events. Consider the two statements :

S1 : $A_1 \cup A_2$ and A_3 are independent

S2 : $A_1 \cap A_2$ and A_3 are independent.

Which of the following holds in general ?

(A) Only S1 is correct

(B) Only S2 is correct

(C) Both S1 and S2 are correct

(D) Neither S1 nor S2 is correct

51. A r.v. X has the probability distribution given by

$$P(X = -1) = \frac{1}{6} = P(X = 4) \text{ and } P(X = 0) = \frac{1}{3} = P(X = 2)$$

Let Y = X/(X + 2). Then E(Y) is :

(A) $\frac{1}{9}$

(B) $\frac{1}{18}$

(C) $\frac{1}{36}$

(D) $\frac{1}{6}$

52. Suppose X is a standard normal r.v. and a and b are constants. Let $Y = (X - a)/b$. Then Y has a non-degenerate normal distribution :

(A) only if a and b are positive (B) for all a, b

(C) for any a but $b \neq 0$ (D) for any a but $b > 0$

53. Let X and Y be two independent r.v.s with common d.f. F and let Z = min (X, Y). Then the d.f. of Z is :

(A) $[1 - F(z)]^2$ (B) $(F(z))^2$

(C) $1 - [1 - F(z)]^2$ (D) $1 - (F(z))^2$

54. If X has uniform distribution over (0, 1), then the distribution of $Y = -\log X$ is :

(A) uniform over $(-\infty, 0)$ (B) uniform over $(0, \infty)$

(C) exponential over $(-\infty, 0)$ (D) exponential over $(0, \infty)$

55. Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. normal r.v.s with $EX_1 = \mu$ and $V(X_1) = \sigma^2 < \infty$. Then the distribution of $Z = \frac{1}{\sigma^2} \sum_1^n (X_i - \mu)^2$ is :

(A) Chi-square with $(n - 1)$ d.f.

(B) Chi-square with $(n - 1)$ d.f. only if $\mu = 0$

(C) Chi-square with n d.f. only if $\sigma = 1$

(D) Chi-square with n d.f.

56. Let $x_1, x_2, \ldots, x_n$ be a random sample from population with p.d.f.

$$f(x, \theta) = \frac{1}{2} e^{|x-\theta|} \quad -\infty < x < \infty, \quad -\infty < \theta < \infty$$

Then MLE of θ is :

(A) sample median

(B) sample mean

(C) the smallest observation $x(1)$

(D) the largest observation $x(n)$

57. In case of Poisson distribution $P(\lambda)$ the Fisher's information contained in a single observation is :

(A) 1 (B) λ

(C) λ^2 (D) λ^{-1}

58. In case of uniform distribution $V(0, \theta)$ the MVUE of θ based on a sample of size n is :

(A) $x_{(n)}$ (B) $x_{(1)} + x_{(n)}$

(C) $\frac{n x_{(n)}}{(n + 1)}$ (D) $\frac{(n + 1)}{n} x_{(n)}$

59. Consider the problem of testing $H_0 : f(x) = f_0(x)$ against $H_1 : f(x) = f_1(x)$ based on a single observation, where

x	0	1	2	3	4
$f_0(x)$	0.1	0.2	0.1	0.4	0.2
$f_1(x)$	0.2	0.2	0.2	0.2	0.2

Consider a test function ϕ, where $\phi(0) = \phi(1) = \phi(2) = 1$ and $\phi(3) = \phi(4) = 0$. If α is the probability of type I error and β is the probability of type II error, then (α, β) is given by :

(A) (0.4, 0.4)
(B) (0.4, 0.6)
(C) (0.6, 0.4)
(D) (0.6, 0.6)

60. Let $\{T_n\}$ be a sequence of unbiased consistent estimators of θ. Then :

(A) $\{T_n^2\}$ is a sequence of biased consistent estimators of θ^2

(B) $\{T_n^2\}$ is a sequence of unbiased consistent estimators of θ^2

(C) $\{T_n^2\}$ is a sequence of unbiased inconsistent estimators of θ^2

(D) None of the above

61. In the regression model $Y = \alpha + \beta X + \epsilon$, where $V(\epsilon) = \sigma^2$, let $\hat{\beta}$ be the least square estimator of β. Then $V(\hat{\beta})$ is :

(A) $(\Sigma X_i^2)\sigma^2$
(B) $(\Sigma X_i^2)^{-1}\sigma^2$
(C) $(\Sigma X_i)\sigma^2$
(D) $(\Sigma X_i)^{-1}\sigma^2$

62. It is proposed to test the hypothesis of symmetry of some unknown distribution with distribution function F on the basis of a random sample $x_1, x_2, \ldots, x_n$ from F. The most appropriate test is :

(A) Mann-Whitney
(B) Chi-square
(C) Kruskal Wallis
(D) Sign test

63. To test the hypothesis $H_0 : \rho = 0$, where ρ is the population correlation coefficient, the appropriate statistic is :

(A) $\dfrac{r}{\sqrt{1-r^2}}\sqrt{n-2}$, where r is the sample correlation coefficient

(B) $\dfrac{r}{\sqrt{1-r^2}}\sqrt{n-3}$

(C) $\dfrac{r\sqrt{n-1}}{\sqrt{1-r^2}}$

(D) None of the above

64. To test the equality of population means of two normal populations with the same unknown variance the following statistic is used $T_n = \frac{\bar{x} - \bar{y}}{s}$, where $\bar{x}, \bar{y}$ are sample means and s a function of the sample observations. Then an appropriate function s is :

(A) $\sqrt{\Sigma(x_i - \bar{x})^2 + \Sigma(y_i - \bar{y})^2}$

(B) $\frac{1}{2}\left[\sqrt{\Sigma(x_i - \bar{x})^2} + \sqrt{\Sigma(y_i - \bar{y})^2}\right]$

(C) $\sqrt{\Sigma(x_i - \bar{z})^2 + \Sigma(y_i - \bar{z})^2}$, where $\bar{z}$ is the pooled sample mean

(D) $\frac{1}{2}\left[\sqrt{\Sigma(x_i - \bar{z})^2} + \sqrt{\Sigma(y_i - \bar{z})^2}\right]$

65. Consider a queueing system $(M|M|1)$ with constant mean arrival rate λ and constant mean service rate μ $(> \lambda)$. Let P_n be the probability that there are n customers in the system. Then :

(A) the system will reach the steady state with $P_n = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n$

(B) the system will reach the steady state with $P_n = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^{n-1}$

(C) the system will not reach steady state at all

(D) the system will reach the steady state but P_n cannot be computed from the given data

66. Consider an inventory problem in which :

(i) demand is uniform at a rate R units per unit time

(ii) lead time is zero

(iii) replenishment is instantaneous

(iv) shortages are not allowed

Suppose Q^* and C^* are the EOQ and the minimum cost respectively. Suppose that the model is improved by allowing shortages and are backlogged. Then :

(A) Q^* increases and C^* also increases

(B) Q^* decreases but C^* increases

(C) both Q^* and C^* decrease

(D) Q^* increases but C^* decreases

67. Suppose that the capital cost of a machine is C, S(t) is its scrap value after t years and $f(t)$ is the maintenance cost at time t, which increases with t. Suppose that the value of money remains same during the period. Then the optimum replacement period n for the machine is given by :

(A) $S(n) = 0$

(B) $f(n) = C$

(C) $S(n) = f(n)$

(D) $f(n)$ = the average annual cost till the date

68. Let P and Q be two LPPs such that one is the dual of the other. Consider the correctness of the following statements when P has no feasible solution.

S1 : Q also has no feasible solution.

S2 : Q has an unbounded optimal feasible solution

Then :

(A) Only S1 is true

(B) Only S2 is true

(C) Either S1 or S2 is true

(D) Neither of them is true

69. Let Q be the dual of an LPP, P. Let P be a maximization problem. Suppose the classes of all feasible solutions of P and Q are denoted by τ_1 and τ_2. Further, let $f(\underline{x})$ and $g(\underline{y})$ be the objective functions in P and Q respectively. Then for $\underline{x} \in \tau_1$ and $\underline{y} \in \tau_2$:

(A) $f(\underline{x}) \le g(\underline{y})$

(B) $f(\underline{x}) \ge g(\underline{y})$

(C) $f(\underline{x}) > g(\underline{y})$

(D) $f(\underline{x}) < g(\underline{y})$

70. Consider a 2 × 2 two person zero sum game without any saddle point and having the pay-off matrix :

Player B

Player A	a_{11}	a_{12}
	a_{21}	a_{22}

Suppose that the optimum mixed strategy for player A is given by (p_1, p_2), where $p_1 + p_2 = 1$. Then p_1/p_2 is given by :

(A) $\dfrac{a_{22} - a_{21}}{a_{11} - a_{12}}$

(B) $\dfrac{a_{22} - a_{12}}{a_{11} - a_{21}}$

(C) $\dfrac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$

(D) None of these

71. A and B drew simple random samples of size 3 from a population of 5 units $\{U_1, U_2, U_3, U_4, U_5\}$. The samples constituted as follows :

$A : \{U_5, U_2, U_4\}$ and $B : \{U_2, U_5, U_2\}$.

Then which of the following statements is *false* ?

(A) A might have drawn by SRSWR

(B) A might have drawn by SRSWOR

(C) B might have drawn by SRSWR

(D) B might have drawn by SRSWOR

72. A sampler has the following rules to follow while conducting a survey using stratified random sampling. In a given stratum take a large sample if :

R1 : the stratum is large

R2 : the stratum is more variable

R3 : sampling is cheaper in the stratum

Should the sampler adopt :

(A) Only R_1

(B) Only R_2

(C) Only R_3

(D) all the three

73. In a factorial experiment concerning 3 factors each at 2 levels, the eight treatment combinations are arranged in 2 blocks in such a way that it was not possible to separate the true interaction (of the 3 factors) from the block difference. Then :

(A) the experiment cannot be analysed

(B) the main effects will contain the additive block effects

(C) the two factor interactions will contain the additive block effects

(D) the three factor interaction and block difference are totally confounded

74. The ANOVA table concerning data from an experiment in a randomised block design is given below. What are the missing elements (x, y) in the table ?

Source of Variation	d.f.	S.S.	M.S.S.
Treatments	4	160	40
Blocks	3	120	40
Error	12	x	30
Total		y	

(A) (360, 640)

(B) (360, 280)

(C) (30, 80)

(D) (30, 640)

Rough Work

Rough Work

SEAL