

Signature of Invigilators

Roll No.

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1 .....

## MATHEMATICAL SCIENCES

(In figures as in Admit Card)

2 .....

### Paper II

Roll No. ....

(In words)

# J—0102

Name of the Areas / Section (if any) .....

Time Allowed : 75 Minutes]

[Maximum Marks : 100

### Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page.
2. This paper consists of *Seventy (70)* multiple choice type questions, out of which any *(50)* are to be attempted.
3. Each item has upto four alternative responses marked (A), (B), (C) and (D). The answer should be a capital letter for the selected option. The answer letter A question should entirely be contained within the corresponding square.

Correct method  A Wrong Method  A or  A

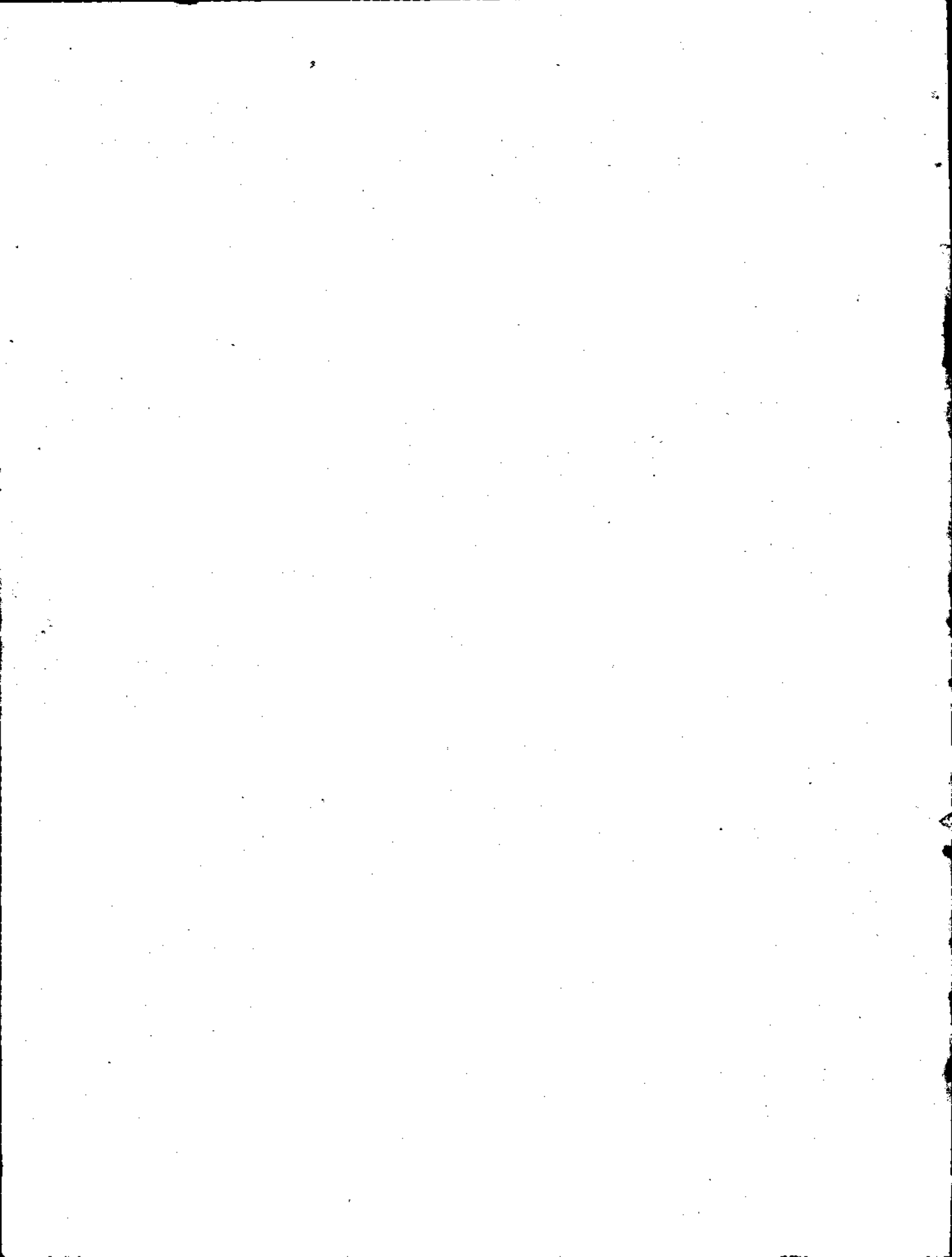
4. Your responses to the items for this paper are to be indicated on the Answer sheet under paper II only.
5. Read instructions given inside carefully.
6. One sheet is attached at the end of the booklet for rough work.
7. You should return the test booklet to the invigilator at the end of paper and should not carry any paper with you outside the examination hall.

પરીક્ષાર્થીઓ માટેની સૂચનાઓ :

૧. આ પાનાની ટોચમાં દર્શાવેલી જગ્યામાં તમારો રોલ નંબર લખો.
૨. આ પ્રશ્નપત્રમાં સીતેર (70) બહુલક્ષી ઉત્તરો ધરાવતા પ્રશ્નો આપેલા છે. જેમાંથી કોઈપણ પચાસ (50) ના ઉત્તરો આપવાના રહેશે.
૩. પ્રત્યેક પ્રશ્ન વધુમાં વધુ ચાર બહુવૈકલ્પિક ઉત્તરો ધરાવે છે. જે (A), (B), (C) અને (D) વડે દર્શાવવામાં આવ્યા છે. પ્રશ્નનો ઉત્તર કેપીટલ સંજ્ઞા વડે આપવાનો રહેશે. ઉત્તરની સંજ્ઞા આપેલ પાનામાં બરાબર સમાઈ જાય તે રીતે લખવાની રહેશે.

ખરી રીત :  A ખોટી રીત :  A ,  A

૪. આ પ્રશ્નપત્રના જવાબ આપેલ Answer Sheet ના Paper II વિભાગની નીચે આપેલ પાનાઓમાં આપવાના રહેશે.
૫. અંદર આપેલ સૂચનાઓ કાળજીપૂર્વક વાંચો.
૬. આ બુક્લેટની પાછળ આપેલું પાનું રફ કામ માટે છે.
૭. પરીક્ષા સમય પૂરો થઈ ગયા પછી આ બુક્લેટ જે તે નીરીક્ષકને સોંપી દેવી. કોઈપણ પેપર પરીક્ષા રૂમની બહાર લઈ જવું નહીં.



## MATHEMATICAL SCIENCES

### PAPER II

Note :—(i) This paper contains *seventy (70)* multiple choice questions, each carrying *two (2)* marks.

(ii) Attempt any *fifty (50)* questions. In case more questions are attempted, only first *fifty (50)* attempted questions will be assessed.

1. Let  $f(x) = x$  ( $x \in \mathbf{R}$ ),  $g(x) = x^2$  ( $x \in \mathbf{R}$ ). Then :
  - (A) both  $f$  and  $g$  are uniformly continuous on  $\mathbf{R}$
  - (B)  $f$  is uniformly continuous on  $\mathbf{R}$  but  $g$  is not
  - (C)  $g$  is uniformly continuous on  $\mathbf{R}$  but  $f$  is not
  - (D) neither  $f$  nor  $g$  is uniformly continuous on  $\mathbf{R}$
2. Let  $\{f_n\}$  be a sequence of real valued functions defined and continuous on  $[0, 1]$ . If  $\{f_n\}$  converges uniformly on  $[0, 1]$  to  $f$ , then :
  - (A)  $f$  is continuous on  $[0, 1]$  but not Riemann integrable on  $[0, 1]$
  - (B)  $f$  is Riemann integrable on  $[0, 1]$ , but not continuous on  $[0, 1]$
  - (C)  $f$  is neither continuous nor Riemann integrable on  $[0, 1]$
  - (D)  $f$  is both continuous and Riemann integrable on  $[0, 1]$
3. Let  $f$  and  $g$  be monotonically increasing on  $[a, b]$ . Then  $f \cdot g$  is :
  - (A) monotonically increasing on  $[a, b]$
  - (B) continuous on  $[a, b]$
  - (C) discontinuous at atmost countable number of points in  $[a, b]$
  - (D) continuous at atmost countable number of points in  $[a, b]$
4. The series  $\sum_{n=0}^{\infty} \frac{z^n}{n!}$  :
  - (A) converges for  $|z| < 1$
  - (B) converges for  $|z| < n!$
  - (C) converges for all  $z$
  - (D) converges only if  $z = 0$

5. The Laurentz series expansion of  $f(z) = \frac{1}{z^2 - 3z + 2}$  in powers of  $(z - 1)$  is :

(A)  $\sum_{n=-1}^{\infty} -(z - 1)^n$

(B)  $\sum_{n=-2}^{\infty} -(z - 1)^n$

(C)  $\sum_{n=0}^{\infty} (z - 1)^n$

(D)  $\sum_{n=0}^{\infty} \frac{1}{(z - 1)^n}$

6. Let  $v_1, v_2, \dots, v_m$  be  $m$  linearly independent column vectors in  $\mathbb{R}^n$ , then the row rank of the  $n \times m$  matrix  $A = [v_1, v_2, \dots, v_m]$  is :

- (A)  $n$                       (B)  $m$                       (C)  $mn$                       (D)  $\max(m, n)$

7. The maximum number of linearly independent characteristic vectors (eigen vectors) of the matrix

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

is :

- (A) 1                      (B) 0                      (C) 2                      (D) 3

8. A square matrix  $A$  is invertible if and only if :

- (A)  $\det A > 0$                       (B)  $\det A < 0$   
(C)  $\det A = \pm 1$                       (D)  $\det A \neq 0$

9. Let  $Q_1$  and  $Q_2$  be two positive definite quadratic forms. Then the following must also be a positive definite quadratic form :
- (A)  $Q_1 Q_2$  (B)  $Q_1 - Q_2$   
 (C)  $Q_1 + Q_2$  (D)  $Q_1/Q_2$
10. If  $P(E) = 1/2$  and  $P(F) = 1/2$ , then :
- (A) E and F are mutually exclusive events  
 (B) E and F are exhaustive events  
 (C) E and F are independent events  
 (D) E and F are not necessarily mutually exclusive events
11. If E and F are independent events, then how many of the following statements are true ?
- $S_1$  :  $E^C$  and F are independent  
 $S_2$  : E and  $F^C$  are independent  
 $S_3$  :  $E^C$  and  $F^C$  are independent
- (A) 0 (B) 1 (C) 2 (D) 3
12. Let X have binomial distribution with parameters  $n$  and  $p$ . Then the distribution of X has a unique mode if :
- (A)  $np$  is an integer  
 (B)  $(n + 1)p$  is an integer  
 (C)  $(n - 1)p$  is an integer  
 (D)  $(n - 1)p$  and  $np$  are both integers
13. For two events E and F, which of the following is false ?
- (A)  $P(E \cup F) \leq P(E) + P(F)$   
 (B)  $P(E \cap F) \leq P(E|F)$   
 (C)  $P(E \cap F) > P(E|F)$   
 (D)  $P(E \cup F) = P(E) + P(F \cap E^C)$

14. A subset is selected at random from  $\{1, 2, \dots, 10\}$ . The probability that it contains the elements 2 and 8 is :
- (A)  $1/4$                       (B)  $1/2$                       (C)  $1/5$                       (D)  $1/6$
15. Two fair dice are thrown simultaneously and the numbers that turned up are denoted by X and Y. Then :
- (A)  $P(X + Y \text{ is an even number}) < P(X + Y \text{ is an odd number})$   
 (B)  $P(X + Y \text{ is an even number}) > P(X + Y \text{ is an odd number})$   
 (C)  $P(X + Y \text{ is an even number}) = P(X + Y \text{ is an odd number})$   
 (D)  $P(X = Y) = 1$
16. Let X be standard normal with p.d.f.  $f$  and Y be normal with mean zero, variance 4 and p.d.f.  $g$ . Then  $f$  and  $g$  :
- (A) do not intersect  
 (B) intersect at 2 points  
 (C) intersect at 4 points  
 (D) intersect at one point only
17. Let  $E_1$  and  $E_2$  be exhaustive and mutually exclusive events with  $P(E_1)P(E_2) > 0$ . Then for any event F, with  $P(F) > 0$ , the correct expression is :

(A) 
$$P(E_2/F) = \frac{P(F/E_1)P(E_1)}{\sum_1^2 P(F/E_i)P(E_i)}$$

(B) 
$$P(E_2/F) = 1 - \frac{P(F/E_1)P(E_1)}{\sum_1^2 P(F/E_i)P(E_i)}$$

(C) 
$$P(F/E_2) = 1 - \frac{P(E_2/F)P(F)}{\sum_1^2 P(E_i/F)P(F)}$$

(D) 
$$P(F/E_2) = \frac{P(E_2/F)P(F)}{\sum_1^2 P(E_i/F)P(F)}$$

18. A system  $Ax = b$ ,  $A : m \times n$  has no unique solution, if :
- (A)  $A$  is non-singular  
 (B)  $A$  is singular and  $b$  is independent relative to the Vectors of  $A$   
 (C)  $A$  is singular  
 (D)  $A$  is singular and  $b$  is dependent relative to the vectors of  $A$
19. For some maximization l.p.p. in  $r$ th iteration of the simplex procedure  $z_j - c_j \geq 0$  for all non-basic variables with  $z_j - c_j = 0$  for at least one non-basic variable. Then the l.p.p. has :
- (A) a unique maximal solution      (B) alternate optimal solution  
 (C) no feasible solution              (D) an unbounded solution
20. If outgoing variable decision is not according to the minimum replacement ratio criterion of the simplex method then the next basic solution will be :
- (A) inferior (non-improving)      (B) infeasible  
 (C) inferior and infeasible      (D) also feasible
21. Let  $E$  be an infinite subset of  $\mathbf{R}$ . Which of the following is true ?
- (A) If  $E$  is uncountable,  $E$  is unbounded  
 (B) If  $E$  is unbounded,  $E$  is uncountable  
 (C) If  $E$  is unbounded and uncountable,  $E = \mathbf{R}$   
 (D) If  $E$  is the range of a non-constant continuous function on  $[0, 1]$ , then  $E$  is uncountable
22. For the series

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^3} + \dots$$

$$\text{let } \liminf_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \alpha \text{ and } \limsup_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \beta.$$

Then :

- (A)  $\alpha = 0, \beta = \infty$                       (B)  $\alpha = \beta = 0$   
 (C)  $\alpha = \beta = \infty$                         (D)  $0 < \alpha < \beta < \infty$

23. Let  $\{a_n\}$  and  $\{b_n\}$  be sequences of real numbers such that  $\{a_n\}$  is Cauchy and  $\{b_n\}$  is convergent. Let  $c_n = a_n + b_n$ . Then  $\{c_n\}$  is :
- (A) convergent (B) not Cauchy  
 (C) Cauchy, but not convergent (D) neither Cauchy nor convergent
24. Let  $\mathbf{R}$ ,  $\mathbf{Q}$ ,  $\mathbf{Z}$  denote the set of real numbers, the set of rational numbers and the set of integers respectively. Then :
- (A)  $\mathbf{Q}$  is ordered, but not a field  
 (B)  $\mathbf{Z}$  is a field, but not ordered  
 (C)  $\mathbf{R}$  is an ordered field but not complete  
 (D)  $\mathbf{R}$  is a complete ordered field
25. Let  $C_1$  be the disc  $|z| < 5/4$  and  $C_2$  be the disc  $|z| < 2$ . The function  $f(z) = z^2 - 3z + 2$  is conformal in :
- (A) both  $C_1$  and  $C_2$  (B)  $C_1$  but not  $C_2$   
 (C)  $C_2$  but not  $C_1$  (D) neither  $C_1$  nor  $C_2$
26. The cross-ratio  $(z_1, z_2, z_3, z_4)$  of four points in the complex plane is real if and only if the four points are :
- (A) the corners of a square (B) on a circle  
 (C) on a straight line (D) on a circle or on a straight line
27. If  $C$  is the circle  $|z| = 10$ , the value of

$$\int_C \frac{dz}{z^2 - 5z + 4} :$$

- (A) does not exist (B) is 0  
 (C) is  $\frac{2}{3} \pi i$  (D) is  $-\frac{2}{3} \pi i$

28. The function

$$f(z) = \frac{\sin 1/z}{z^2 + 11z + 13}$$

has :

- (A) no singularities  
 (B) only poles  
 (C) only an essential singularity  
 (D) both an essential singularity and poles



29. For elements  $a$  and  $b$  of a group,  $a^{-1}b$  is an element of order 2. What is the order of  $ba^{-1}$  ?
- (A) 2  
 (B) Products of the orders of  $a$  and  $b$   
 (C) l.c.m. of the orders of  $a$  and  $b$   
 (D) Any number
30. Let  $G = \{f_{a,b} \mid a, b \text{ real, } a \neq 0\}$ , where  $f_{a,b}(x) = ax + b$  for real  $x$ . Consider the group  $G$  under the composition of mappings. What is the inverse of  $f_{2,4}$  ?
- (A)  $f_{4,2}$                       (B)  $f_{2,1/4}$                       (C)  $f_{1/4,2}$                       (D)  $f_{1/2,(-2)}$
31. Let  $G$  be the multiplicative group of non-zero complex numbers and  $H$  the subgroup of complex numbers with modulus 1. Which of the following is isomorphic to  $G/H$  ?
- (A)  $G$   
 (B)  $H$   
 (C) The additive group of all real numbers  
 (D) The multiplicative group of all positive real numbers
32. The subgroup of  $S_3$ , which is cyclic of order 3, is generated by :
- (A) (12)                      (B) (23)                      (C) (13)                      (D) (132)
33. Let  $R[x]$  be the ring of all polynomials with real coefficients. Which of the following subrings of  $R[x]$  is *not* an ideal of  $R[x]$  ?
- (A)  $\{f(x) \mid f(0) = 0\}$                       (B)  $\{f(x) \mid f(0) = 0 = f'(0)\}$   
 (C)  $\{f(x) \mid f(1) = 0 = f(3)\}$                       (D)  $\{f(x) \mid \text{Degree of } f = 0\}$
34. Let  $W_1$  and  $W_2$  be subspaces of a vector space  $V$  such that  $W_1 \cap W_2 = \{0\}$ . Let  $w_1 \in W_1$  and  $w_2 \in W_2$  be non-zero vectors. Which of the following is true ?
- (A)  $\{w_1, w_2\}$  and  $\{w_1 + w_2, w_1 - w_2\}$  are both linearly independent sets.  
 (B)  $\{w_1, w_2\}$  is linearly independent but  $\{w_1 + w_2, w_1 - w_2\}$  is a linearly dependent set.  
 (C)  $\{w_1, w_2\}$  is a linearly dependent set while  $\{w_1 + w_2, w_1 - w_2\}$  is linearly independent.  
 (D)  $\{w_1, w_2\}$  and  $\{w_1 + w_2, w_1 - w_2\}$  are both linearly dependent sets.

35. Consider the linear transformation  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  given by the matrix

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Which of the following is a *true* statement ?

- (A)  $T$  is one-one and onto      (B)  $T$  is onto but not one-one  
 (C)  $T$  is one-one but not onto      (D)  $T$  is neither one-one nor onto
36. Let  $T : V \rightarrow W$  be a linear map between finite dimensional vector spaces  $V$  and  $W$ . Let  $T' : W' \rightarrow V'$  be the dual map. Which of the following is a *true* statement ?
- (A)  $T$  is surjective if and only if  $T'$  is injective  
 (B)  $T$  is surjective if and only if  $T'$  is surjective  
 (C)  $T$  is injective if and only if  $T'$  is injective  
 (D) Injectivity or surjectivity of  $T$  and  $T'$  are unrelated
37. Let  $V$  be an inner product space and  $P : V \rightarrow V$  be a linear map. Then  $P$  is an orthogonal projection if and only if :
- (A)  $P^2 = -P$  and  $P^* = P$       (B)  $P^2 = P$  and  $P^* = -P$   
 (C)  $P^2 = P$  and  $(I - P)^2 = I - P$       (D)  $P^2 = P$  and  $P^* = P$

[Here  $P^*$  denotes the adjoint of  $P$ .]

38. The linear map  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  described by

$$T(\hat{i}) = \hat{j} + \hat{k}, T(\hat{j}) = \hat{k} + \hat{i}, T(\hat{k}) = \hat{i} + \hat{j}$$

has the following matrix representation :

(A)  $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

(B)  $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(C)  $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

(D)  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

39. The initial value problem

$$x \frac{dy}{dx} = y + 1, y(0) = -1$$

has :

- (A) a unique solution
  - (B) more than one but finitely many solutions
  - (C) infinitely many solutions
  - (D) no solution
40. For the differential equation

$$\frac{dx}{dt} + \beta^2 x = 0,$$

where  $\beta$  is a non-zero real constant :

- (A) all solutions become unbounded as  $t \rightarrow \infty$ , when  $\beta < 0$
  - (B) all solutions, with one exception, become unbounded as  $t \rightarrow \infty$ , when  $\beta < 0$
  - (C) all solutions, with one exception, approach zero as  $t \rightarrow \infty$  when  $\beta > 0$
  - (D) all solutions approach zero as  $t \rightarrow \infty$
41. Let  $y_1(x)$  and  $y_2(x)$  be two solutions of  $y'' + 3x^2y' + e^{-x^2}y = 0$ , satisfying  $y_1(0) = 0$ ,  $y_1'(0) = -1$ ,  $y_2(0) = 1$ ,  $y_2'(0) = -1$ . Then the Wronskian  $W(y_1, y_2)(x)$  of  $y_1(x)$  and  $y_2(x)$  at  $x = 1$  is :
- (A)  $e$
  - (B)  $e^{-1}$
  - (C)  $-e$
  - (D)  $-e^{-1}$
42. The initial value problem  $y'' + x^2y' + e^{-x}y = \sin x$ ,  $0 < x < 1$ ,  $y(0) = 0$ ,  $y'(0) = 1$  has :
- (A) a unique solution on  $0 < x < 1$
  - (B) more than one, but finitely many solutions on  $0 < x < 1$
  - (C) infinitely many solutions on  $0 < x < 1$
  - (D) no solution on  $0 < x < 1$

43. The characteristic curves of the partial differential equation

$$\frac{\partial z}{\partial y} + y \frac{\partial z}{\partial x} = \frac{z}{y}, \quad y \neq 0$$

are :

- (A) straight lines along which  $z$  varies proportional to  $x$
  - (B) circles along which  $z$  remains constant
  - (C) parabolas along which  $z$  varies proportional to  $y$
  - (D) exponential curves along which  $z$  satisfies an ordinary differential equation
44. If  $u(x, y)$  satisfies the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} - u = 0,$$

and  $A, B$  are arbitrary functions of a single variable, either  $x$  or  $y$ , then :

- (A)  $u = A(x) e^x + B(y) e^{-x}$
  - (B)  $u = A(y) e^x + B(x) e^{-x}$
  - (C)  $u = A(x) e^x + B(x) e^{-x}$
  - (D)  $u = A(y) e^x + B(y) e^{-x}$
45. In the theory of least square method of estimating the unknown parameters of the model, we minimize the sum of :
- (A) squares of observations
  - (B) observations
  - (C) deviations of observations from their expected values
  - (D) square of the deviations of observations from their expected values

46. If  $B_{YX} = 0.4$  and  $B_{XY} = 0.16$  are the regression coefficients of  $Y$  on  $X$  and  $X$  on  $Y$  respectively, then the correlation coefficient between  $X$  and  $Y$  is :

- (A) 0.08
- (B) -0.08
- (C) 0.0064
- (D) None of these

47. If the joint distribution of  $X$  and  $Y$  is given by the p.d.f.  $f(x, y)$ , conditional expectation of  $Y$  given  $X = x$  is :

- (A)  $\int \frac{y f(x, y) dy}{\int f(x, y) dy}$
- (B)  $\int \frac{y f(x, y) dy}{\int f(x, y) dx}$
- (C)  $\int \frac{y f(x, y) dy}{P(X = x)}$
- (D)  $\int y f(x, y) dy$

48. Let  $X$  be absolutely continuous with p.d.f.  $f(x)$ . Then  $Y = X^2$  has density :

(A)  $f(\sqrt{y}) \cdot \frac{1}{2\sqrt{y}}$

(B)  $[f(\sqrt{y}) + f(-\sqrt{y})] \frac{1}{2\sqrt{y}}$

(C)  $[f(\sqrt{y}) - f(-\sqrt{y})] \frac{1}{2\sqrt{y}}$

(D)  $f(-\sqrt{y}) \frac{1}{2\sqrt{y}}$

49. The characteristic function of uniform distribution over  $(-\alpha, \alpha)$  is :

(A)  $\frac{1}{\alpha t} \sin(t\alpha)$

(B)  $\frac{1}{\alpha t} \cos(t\alpha)$

(C)  $\alpha t \sin(t\alpha)$

(D)  $\alpha t \cos(t\alpha)$

50. If  $X_1, X_2, \dots, X_n$  are i.i.d. r.v.s. with  $EX_1 = \mu$  and  $V(X_1) = \sigma^2$ , then which of the following is a consequence of Chebyshev's inequality ?

(A)  $P[|\bar{X} - \mu| > c\sigma^2] \leq 1/c^2$

(B)  $P[|\bar{X} - \mu| > c\sigma] \leq 1/c^2$

(C)  $P[|\bar{X} - \mu| < c\sigma] \leq 1/c^2$

(D)  $P[|\bar{X} - \mu| > c\sigma] > 1 - 1/c^2$

51. Let  $X$  have a rectangular distribution over  $(0, \theta)$ . Then :

(A) The variance of  $X <$  the mean of  $X$

(B) The variance of  $X >$  the mean of  $X$

(C) The variance of  $X =$  the mean of  $X$

(D) Nothing can be said about the mean and variance of  $X$  in general

52. If  $X$  is a r.v. with  $E X = \text{variance of } X$ , then :

- (A)  $X$  is a Poisson r.v.
- (B)  $X$  is a Uniform r.v.
- (C)  $X$  is a binomial r.v.
- (D) Nothing can be said about the distribution of  $X$

53. Let  $X_1, X_2, \dots, X_n$  be i.i.d. standard normal r.v.s. Consider :

$$Y_1 = \frac{\bar{X}\sqrt{n-1}}{\sqrt{\sum_i^n (X_i - \bar{X})^2}} \quad \text{and} \quad Y_2 = \frac{\bar{X}\sqrt{n}}{\sqrt{\sum_1^n (X_i - \bar{X})^2}}$$

Then :

- (A) Both  $Y_1$  and  $Y_2$  are distributed as Student's  $t$  with  $(n - 1)$  d.f.
  - (B) Both  $Y_1$  and  $Y_2$  are distributed as Student's  $t$  with  $n$  d.f.
  - (C)  $Y_1$  is distributed as Student's  $t$  with  $(n - 1)$  d.f. but  $Y_2$  is not distributed as Student's  $t$  with  $n$  d.f.
  - (D)  $Y_2$  is distributed as Student's  $t$  with  $(n - 1)$  d.f. but  $Y_1$  is not distributed as Student's  $t$  with  $n$  d.f.
54. If  $X$  is binomial with variance  $\beta$  and

$$P(X = 1) = \alpha P(X = 0)$$

then :

- (A)  $E X = \alpha\beta$
  - (B)  $E X = \alpha/\beta$
  - (C)  $E X = \beta/\alpha$
  - (D)  $E X = \sqrt{\alpha\beta}$
55. When a hypothesis  $H_0$  Vs.  $H_1$  is tested, which of the following is the *correct* interpretation ?
- (A) The smaller the  $P$  value, the stronger is the evidence against  $H_0$  provided by the data
  - (B) The smaller the  $P$  value, the stronger is the evidence in favour of  $H_0$
  - (C) The larger the  $P$  value the stronger is the evidence against  $H_0$
  - (D)  $P$  value is always given before the test and hence has no relevance for or against  $H_0$

56. Let  $X$  be an observation from  $B(n, p)$  where  $p \in [0.2, 0.8]$ . Then the MLE of  $p$  is :
- (A)  $0.6X + 0.2$  (B)  $0.2X + 0.6$  (C)  $0.2X + 0.8$  (D)  $0.8X + 0.2$
57. Let  $T_1$  and  $T_2$  be two unbiased estimators of  $\theta$ , where  $T_1$  is not sufficient but  $T_2$  is sufficient. Let  $h(T_2) = E(T_1/T_2)$ . Then :
- (A)  $h(T_2)$  is better than  $T_1$   
 (B)  $h(T_2)$  is not better than  $T_1$   
 (C)  $h(T_2)$  is as good as  $T_1$   
 (D)  $h(T_2)$  and  $T_1$  are not comparable
58. Let  $X_1$  and  $X_2$  be i.i.d. standard normal r.v.s. Then  $T = X_1$  is :
- (A) sufficient (B) complete but not sufficient  
 (C) sufficient but not complete (D) both sufficient and complete
59. Consider the assertions :
- (a<sub>1</sub>) In analysis of two-way classified data it is necessary to have equal number of observations per cell.  
 (a<sub>2</sub>) The model will lose orthogonality when the numbers of observations per cell are not equal.
- Choose your answer from the following :
- (A) Both (a<sub>1</sub>) and (a<sub>2</sub>) are correct and (a<sub>2</sub>) is the correct explanation of (a<sub>1</sub>)  
 (B) Both (a<sub>1</sub>) and (a<sub>2</sub>) are correct and (a<sub>2</sub>) is not the correct explanation of (a<sub>1</sub>)  
 (C) (a<sub>1</sub>) is true but (a<sub>2</sub>) is false  
 (D) (a<sub>1</sub>) is false but (a<sub>2</sub>) is true
60. For testing  $H_0 : \mu_1 = \mu_2$  against  $H_1 : \mu_1 \neq \mu_2$  based on two independent random samples of size  $n_1$  and  $n_2$  drawn from  $N(\mu_1, \sigma_1^2)$  and  $N(\mu_2, \sigma_2^2)$  respectively, we use the following test when  $\sigma_1^2$  and  $\sigma_2^2$  are unknown :
- (A) Student's  $t$  test (B) Standard normal deviate test  
 (C) Chi-square test (D) Fisher-Behren test

61. Patients arrive at a getwell clinic of a Dr. XYZ according to a Poisson process at the rate 20 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient for Dr. XYZ is exponential, with a mean of 8 minutes.

The queuing model for the problem is :

- (A) M/M/3 (B) M/M/3 with finite capacity  
 (C) M/M/1 (D) M/M/1 with finite capacity
62. For the M/M/1/FIFO/N model, in usual notations,  $\lambda_{\text{eff}}$  (effective arrival rate of the system) is given by :
- (A)  $\mu(L_s - L_q)$  (B)  $\mu(L_q - L_s)$  (C)  $\lambda(L_s - L_q)$  (D)  $\lambda(L_q - L_s)$
63. If K is set up cost, R is uniform demand rate, h is holding cost per unit per unit time, a is the cost of a unit then for an EOQ model under the assumption of instantaneous production, the EOQ is :

(A)  $\sqrt{\frac{2KR}{h}}$

(B)  $\sqrt{\frac{2KR}{h\left(1 - \frac{R}{a}\right)}}$

(C)  $\sqrt{\frac{2KR}{ha}}$

(D) None of the above

64. Below given is a transportation problem model :

Supply :  $a_1 = 10, a_2 = 5, a_3 = 14, a_4 = 16$

Demand :  $b_1 = 10, b_2 = 13, b_3 = 12, b_4 = 4, b_5 = 3$

To balance it we need to add :

- (A) a dummy source  
 (B) a dummy destination  
 (C) both a dummy source and a dummy destination  
 (D) neither a dummy source nor a dummy destination



65. In a cost minimization Assignment Problem (A.P.) if  $p_i$  and  $q_j$  represent  $i$ th row minima and  $j$ th column minima respectively then the minimum cost of the A.P. is :

- (A)  $\sum_i p_i + \sum_j q_j$   
 (B) more than  $\sum_i p_i + \sum_j q_j$   
 (C) less than  $\sum_i p_i + \sum_j q_j$   
 (D)  $\sum_i p_i - \sum_j q_j$

66. Consider the following  $2 \times 4$  games. The pay off for player A is :

		Player B			
		B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>
Player A	A <sub>1</sub>	2	2	3	-1
	A <sub>2</sub>	4	3	2	6

When solved graphically, the optimum solution for player A is given by :

- (A) maximum of upper envelop  
 (B) maximum of lower envelop  
 (C) minimax of upper envelop  
 (D) maximin of lower envelop
67. The estimator of population mean based on a simple random sample allowing repetition of units is :
- (A) always finitely consistent  
 (B) always finitely inconsistent  
 (C) finitely consistent depending on the nature of population  
 (D) finitely consistent only if they are unbiased

68. Let  $\bar{y}_{sy}$  be the mean of a systematic sample of size  $n$  and  $\bar{y}_{SRS}$  be the mean of a SRS sample of size  $n$ , then :

(A)  $V(\bar{y}_{sy}) < V(\bar{y}_{SRS})$

(B)  $V(\bar{y}_{sy}) > V(\bar{y}_{SRS})$

(C)  $V(\bar{y}_{sy}) = V(\bar{y}_{SRS})$

(D)  $V(\bar{y}_{sy}) < V(\bar{y}_{SRS})$  if intraclass correlation coefficient is negative

69. If C-matrix of an RBD with  $b$  blocks and  $r$  treatments is  $xI_v + yJ_{vv}$ , where  $I_v$  is  $v \times v$  identity matrix and  $J_{vv}$  is  $v \times v$  matrix of unities, then  $x$  and  $y$  are given by :

(A)  $x = v, y = r/v$

(B)  $x = v, y = v/r$

(C)  $x = b, y = -b/v$

(D)  $x = b, y = b/v$

70. Confounding NPK in a  $2^3$  experiment, the layout in a single replication is as follows :

$$B_1 : (1) \quad pk \quad nk \quad np$$

$$B_2 : n \quad k \quad npk \quad p$$

Here the contrast carried by NPK is given by :

(A)  $B_1 - B_2$

(B)  $B_1 + B_2$

(C)  $B_2 - B_1$

(D) None of the above

**ROUGH WORK**

**ROUGH WORK**

**SEAL**